MATH 11008 | Explorations in Modern Mathematics | Fall 2014

Circle one: 9:55 / 12:05 | Dr. Kracht

Print Name: KEY

Exam 3 Version B

Friday, October 24, 2014

NO CALCULATORS.

Academic Honesty Pledge

Your signature at the bottom indicates your agreement to abide by the following rules.

1. I have placed all purses, bags, books, notes, and other papers in the designated area of the classroom.
2. **I have placed all cell phones, calculators, and other electronic devices in the designated area of the classroom.**
3. I will not communicate with other students during the exam.
4. I will not seek help from or give help to others during the exam.
5. I will turn my exam in and will not take it from the classroom.
6. I will not discuss the exam outside of class with another student who has not yet taken the exam.
7. I will not cheat in any other way.
8. I will follow any other instructions from my professor.

Signature: _____.

Good Luck!

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Part I: Long Answer.

No credit for answers without sufficient justification. Use standard mathematical notation correctly.

1. (20 points) Evaluate each of the following by hand. Write each answer as a whole number or fraction in lowest terms. *No credit for answers without all steps written out clearly.*

(a) $\frac{821!}{820!} = \frac{821 \cdot 820!}{820!}$

$= 821.$

(b) $\frac{7!}{7} = \frac{7 \cdot 6!}{7}$

$= 6!$

$= 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

$= 720.$

(c) $\frac{11!}{7!\,4!} = \frac{11 \cdot 10 \cdot 9 \cdot 8 \cdot (7!)}{(7!)(4 \cdot 3 \cdot 2 \cdot 1)}$

$= \frac{11 \cdot 10 \cdot \overset{3}{9} \cdot 8}{4 \cdot \cancel{3} \cdot \cancel{2} \cdot 1}$

$= 11 \cdot 10 \cdot 3$

$= 33 \cdot 10$

$= 330.$

(d) ${}_9C_2 = \frac{9!}{(9-2)!\,2!}$

$= \frac{9!}{7!\,2!}$

$= \frac{9 \cdot 8 \cdot (7!)}{(7!) \cdot 2 \cdot 1}$

$= \frac{9 \cdot 8}{2}$

$= 9 \cdot 4$

$= 36.$

(e) ${}_9P_2$

${}_9P_2 = \frac{9!}{(9-2)!}$

$= \frac{9!}{7!}$

$= \frac{9 \cdot 8 \cdot (7!)}{(7!)}$

$= 9 \cdot 8$

$= 72.$

Version B

2. (25 points) Consider an six-sided die (with sides labeled "1," "2," ..., "6") and a four-sided die (with sides labeled "1," "2," "3," and "4"). The dice are tossed and we observe the number that comes up on each die.

   (a) Write out the sample space S for this random experiment completely. Record each outcome as an ordered pair of the form (s, f) where s is the number that comes up on the six-sided die and f is the number that comes up on the four-sided die.

$S = \{(1,1), (1,2), (1,3), (1,4),$
$(2,1), (2,2), (2,3), (2,4),$
$(3,1), (3,2), (3,3), (3,4),$
$(4,1), (4,2), (4,3), (4,4),$
$(5,1), (5,2), (5,3), (5,4),$
$(6,1), (6,2), (6,3), (6,4)\}$

   (b) Let E_1 be the event "roll doubles or a total of six."

      i. Write E_1 as a set.

$E_1 = \{(1,1), (2,2), (3,3), (4,4), (2,4), (4,2), (5,1)\}$

      ii. Find $|E_1|$. $= 7$

      iii. Find $\Pr(E_1)$. $= \frac{7}{24}$

   (c) Let E_2 be the event "roll a total of twelve."

      i. Write E_2 as a set.

$E_2 = \{\}$

      ii. Find $|E_2|$. $= 0$

      iii. Find $\Pr(E_2)$. $= \frac{0}{24} = 0$.

Part II: Multiple Choice (5 points each)

Circle the letter of the best answer.

3. Which of the following is not a random experiment?

E

(a) drawing a card from a deck of playing cards
(b) tossing a coin three times
(c) selecting numbers in a lottery
(d) rolling a pair of dice
(e) All of these are random experiments

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4. In the game of Bunco, three ordinary six-sided dice are rolled and the number on each die is observed. What is the size of the sample space?

$6 \cdot 6 \cdot 6 = 6^3$

C

(a) $6!$
(b) ${}_6C_3$
(c) 6^3
(d) 3^6
(e) none of these

---

5. A person shoots 4 consecutive free throws and the total number of successes is observed. What is the size of the sample space?

B

$S = \{0, 1, 2, 3, 4\}$

$|S| = 5$

(a) $4!$
(b) 5
(c) 4^2
(d) 2^4
(e) none of these

---

6. Johnie's Coffee Shop offers platters consisting of a sandwich, salad, vegetable, and drink. There are eight different sandwiches, three salads, five vegetables, and ten drinks to choose from. How many different platters does Johnie's offer?

C

$8 \cdot 3 \cdot 5 \cdot 10 = 40 \cdot 3 \cdot 10$
$= 120 \cdot 10$
$= 1200$

(a) 26
(b) 130
(c) 1200
(d) 2000
(e) none of these

7. Eight children are trick-or-treating together. In how many ways can they line up at the door to receive their treats?

A (a) $8!$ $\quad 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 8!$

(b) ${}_8\mathrm{C}_8$

(c) 8

(d) 2^8

(e) none of these

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8. Twenty contestants are entered in a Halloween costume contest. In how many ways can one choose the top four finishers regardless of order? combination

A (a) ${}_{20}\mathrm{C}_4$ $\quad {}_{20}C_4$

(b) $4!$

(c) 20^4

(d) ${}_{20}\mathrm{P}_4$

(e) none of these

---

9. Twenty contestants are entered in a Halloween costume contest. In how many ways can one choose Best Costume and the first-, second-, and third-runners-up? permutation

D (a) ${}_{20}\mathrm{C}_4$ $\quad {}_{20}P_4$

(b) $4!$

(c) 20^4

(d) ${}_{20}\mathrm{P}_4$

(e) none of these

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10. In a probability space, which of the following statements is not necessarily true?

D (a) The probability of the sample space is equal to 1.

(b) The probability of an outcome is always between 0 and 1 (inclusive).

(c) The probability of the impossible event is 0.

(d) All outcomes are equally likely.

(e) All of these statements are always true.

Let $\Pr(o_4) = p$

11. Consider the sample space $S = \{o_1, o_2, o_3, o_4\}$. Suppose $\Pr(o_1) = 0.30$ and $\Pr(o_2) = 0.50$. If o_4 is four times as likely as o_3, find $\Pr(o_4)$.

B

(a) 0.10
(b) 0.16
(c) 0.20
(d) 0.40
(e) none of these

$\Pr(o_4) = 4\Pr(o_3) = 4p$

We always have

$\Pr(o_1) + \Pr(o_2) + \Pr(o_3) + \Pr(o_4) = 1$

$0.30 + 0.50 + p + 4p = 1$

$0.80 + 5p = 1$

$5p = 1 - 0.80$

$5p = 0.20$

$p = 0.04$

So $\Pr(o_4) = 4p = 4(0.04) = 0.16$

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12. An honest coin is tossed five times. What is the probability of tossing 2 heads and 3 tails?

D

(a) $\frac{{}_5P_2}{2^5}$
(b) $\frac{2}{2^5}$
(c) $\frac{{}_5C_2}{5!}$
(d) $\frac{{}_5C_2}{2^5}$
(e) none of these

$|S| = 2^5$

event E

$|E| = {}_5C_2$ ← # ways to choose the positions of the 2 heads

So $\Pr(E) = \frac{|E|}{|S|} = \frac{{}_5C_2}{2^5}$

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13. In how many ways can one distribute 3 ghost costumes, 5 vampire costumes, and 2 witch costumes among ten children?

A

(a) $({}_{10}C_3) \cdot ({}_7C_5) \cdot ({}_2C_2)$
(b) $({}_{10}P_3) \cdot ({}_7P_5) \cdot ({}_2P_2)$
(c) $3! + 5! + 2!$
(d) $({}_{10}C_3) \cdot ({}_{10}C_5) \cdot ({}_{10}C_2)$
(e) none of these

${}_{10}C_3$: # ways to choose ghosts;
then there are 7 children left:
${}_7C_5$: # ways to choose vampires;
then there are 2 children left:
${}_2C_2 = 1$: # ways to choose witches

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14. How many permutations are there of the letters in the word "HALLOWEEN"?

C

(a) $({}_9P_2) \cdot ({}_7P_2) \cdot ({}_5P_5)$
(b) $({}_9C_2) \cdot ({}_7C_2) \cdot ({}_5C_5)$
(c) $({}_9C_2) \cdot ({}_7C_2) \cdot (5!)$
(d) $9!$
(e) none of these

First pick the positions of the 2 L's:
${}_9C_2$
Next choose positions of the 2 E's:
${}_7C_2$
Then put the remaining 5 letters in order:
${}_5P_5 = 5!$