

Quiz 2

For each conjecture, indicate whether it is true or false. If true, give a proof (a logical argument in paragraph form). If false, give a counter example (a specific graph for which it fails, with explanation).

1. **Josh's and Jaden's Conjecture:** *If there are the same number or more odd than even degree vertices in a graph, then there is no Euler path.*

(a) Circle one: TRUE or FALSE

(b) Proof or counter example:

Here is one counter example. (There are many others.)



Degrees of vertices: A:1
 B:2
 C:1

This graph has two odd vertices, but only one even vertex. However, ABC is an Euler Path.

2. **Alex's and Jess's Conjecture:** *If there are more than two vertices of odd degree in a graph, then there is no Euler path.*

(a) Circle one: TRUE or FALSE

(b) Proof or counter example:

Proof. Consider a vertex X in the interior of an Euler Path. (That is, X is not the start or the end of the path.) Every time X is visited on this path, we come into X on one edge and go out of X on another. This way the edges adjacent to vertex X come in pairs. Thus every vertex in the interior of an Euler Path has even degree. Therefore, there cannot be more than two odd vertices in a graph with an Euler Path. \blacksquare