MATH 11008 | Explorations in Modern Mathematics | Fall 2014

Circle one: 9:55 / 12:05 | Dr. Kracht

Name: KEY Quiz Score: /20

Quiz 5: Version A

No credit for answers without sufficient justification. Use standard mathematical notation correctly.
NO CALCULATORS ALLOWED.

1. (14 pts) Eleven web sites are entered in a web design contest. *For each of the following, first give your answer in symbolic form ($_nP_r$ or $_nC_r$), then evaluate without using a calculator.*

   (a) In how many ways can one choose the top three sites regardless of order? combinations

$$_{11}C_3 = \frac{11!}{(11-3)!\,3!} = \frac{11!}{8!\,3!} = \frac{11\cdot 10\cdot 9\,(8!)}{(8!)(3\cdot 2\cdot 1)} = \frac{11\cdot 10^{5}\cdot 9^{3}}{3\cdot 2} = 11\cdot 15 = 165.$$

$$\begin{array}{r} 15 \\ 11 \\ \hline 15 \\ 15 \\ \hline 165 \end{array}$$

   (b) In how many ways can one choose the first-, second-, and third-place winners? permutations

$$_{11}P_3 = \frac{11!}{(11-3)!} = \frac{11!}{8!} = \frac{11\cdot 10\cdot 9\,(8!)}{8!} = 11\cdot 10\cdot 9 = 99\cdot 10 = 990.$$

2. (6 pts) Consider the sample space $S = \{o_1, o_2, o_3, o_4\}$. Suppose $\Pr(o_1) = 0.30$ and $\Pr(o_2) = 0.26$. If o_3 and o_4 are equally likely, complete the probability assignment for this probability space.

We know that $\Pr(o_3) = \Pr(o_4)$ and that

$$\Pr(o_1) + \Pr(o_2) + \Pr(o_3) + \Pr(o_4) = 1$$
$$0.30 + 0.26 + 2\Pr(o_3) = 1$$
$$2\Pr(o_3) = 1 - 0.56$$
$$2\Pr(o_3) = 0.44$$
$$\Pr(o_3) = 0.22$$

$\Pr(o_1) = 0.30$ $\Pr(o_2) = 0.26$ $\Pr(o_3) =$ 0.22 $\Pr(o_4) =$ 0.22

MATH 11008 | Explorations in Modern Mathematics | Fall 2014

Circle one: 9:55 / 12:05 | Dr. Kracht

Name: KEY Quiz Score: /20

Quiz 5: Version B

No credit for answers without sufficient justification. Use standard mathematical notation correctly.
NO CALCULATORS ALLOWED.

1. (14 pts) Nine web sites are entered in a web design contest. *For each of the following, first give your answer in symbolic form ($_nP_r$ or $_nC_r$), then evaluate without using a calculator.*

   (a) In how many ways can one choose the top three sites regardless of order? combinations

$$_9C_3 = \frac{9!}{(9-3)!3!} = \frac{9!}{6!3!} = \frac{9\cdot 8\cdot 7\,(6!)}{(6!)(3\cdot 2\cdot 1)} = \frac{3\,9\cdot 8^{4}\cdot 7}{3\cdot 2\cdot 1} = 3\cdot 4\cdot 7 = 84.$$

   (b) In how many ways can one choose the first-, second-, and third-place winners? permutations

$$_9P_3 = \frac{9!}{(9-3)!} = \frac{9!}{6!} = \frac{9\cdot 8\cdot 7\,(6!)}{6!} = 9\cdot 8\cdot 7 = 504.$$

$$\begin{array}{r} 5 \\ 56 \\ \times\ 9 \\ \hline 504 \end{array}$$

2. (6 pts) Consider the sample space $S = \{o_1, o_2, o_3, o_4\}$. Suppose $\Pr(o_1) = 0.12$ and $\Pr(o_2) = 0.26$. If o_3 and o_4 are equally likely, complete the probability assignment for this probability space.

We know that $\Pr(o_3) = \Pr(o_4)$ and that

$$\Pr(o_1) + \Pr(o_2) + \Pr(o_3) + \Pr(o_4) = 1$$
$$0.12 + 0.26 + 2\Pr(o_3) = 1$$
$$2\Pr(o_3) = 1 - 0.38$$
$$2\Pr(o_3) = 0.62$$
$$\Pr(o_3) = 0.31$$

$\Pr(o_1) = 0.12$ $\Pr(o_2) = 0.26$ $\Pr(o_3) =$ 0.31 $\Pr(o_4) =$ 0.31

MATH 11008 | Explorations in Modern Mathematics | Fall 2014

Circle one: 9:55 / 12:05 | Dr. Kracht

Name: KEY Quiz Score: /20

Quiz 5: Version C

No credit for answers without sufficient justification. Use standard mathematical notation correctly.
NO CALCULATORS ALLOWED.

1. (14 pts) Eight web sites are entered in a web design contest. *For each of the following, first give your answer in symbolic form ($_nP_r$ or $_nC_r$), then evaluate without using a calculator.*

   (a) In how many ways can one choose the top three sites regardless of order? combinations

$$_8C_3 = \frac{8!}{(8-3)!\,3!} = \frac{8!}{5!\,3!} = \frac{8\cdot 7\cdot 6\,(5!)}{(5!)(3\cdot 2\cdot 1)} = \frac{8\cdot 7\cdot 6}{3\cdot 2} = 8\cdot 7 = 56.$$

   (b) In how many ways can one choose the first-, second-, and third-place winners? permutations

$$_8P_3 = \frac{8!}{(8-3)!} = \frac{8!}{5!} = \frac{8\cdot 7\cdot 6\,(5!)}{5!} = 8\cdot 7\cdot 6 = 336.$$

2. (6 pts) Consider the sample space $S = \{o_1, o_2, o_3, o_4\}$. Suppose $\Pr(o_1) = 0.21$ and $\Pr(o_2) = 0.43$. If o_3 and o_4 are equally likely, complete the probability assignment for this probability space.

We know that $\Pr(o_3) = \Pr(o_4)$ and that

$$\Pr(o_1) + \Pr(o_2) + \Pr(o_3) + \Pr(o_4) = 1$$
$$0.21 + 0.43 + 2\Pr(o_3) = 1$$
$$2\Pr(o_3) = 1 - 0.64$$
$$2\Pr(o_3) = 0.36$$
$$\Pr(o_3) = 0.18$$

$\Pr(o_1) = 0.21$ $\Pr(o_2) = 0.43$ $\Pr(o_3) =$ 0.18 $\Pr(o_4) =$ 0.18

MATH 11008 | Explorations in Modern Mathematics | Fall 2014

Circle one: 9:55 / 12:05 | Dr. Kracht

Name: KEY Quiz Score: /20

Quiz 5: Version D

No credit for answers without sufficient justification. Use standard mathematical notation correctly.
NO CALCULATORS ALLOWED.

1. (14 pts) Ten web sites are entered in a web design contest. *For each of the following, first give your answer in symbolic form ($_nP_r$ or $_nC_r$), then evaluate without using a calculator.*

   (a) In how many ways can one choose the top three sites regardless of order? Combinations

$$_{10}C_3 = \frac{10!}{(10-3)!\,3!} = \frac{10!}{7!\,3!} = \frac{10\cdot 9\cdot 8\cdot (7!)}{(7!)(3\cdot 2\cdot 1)} = \frac{10\cdot 9^{3}\cdot 8^{4}}{3\cdot 2} = 10\cdot 3\cdot 4 = 10\cdot 12 = 120.$$

   (b) In how many ways can one choose the first-, second-, and third-place winners? permutations

$$_{10}P_3 = \frac{10!}{(10-3)!} = \frac{10!}{7!} = \frac{10\cdot 9\cdot 8\cdot (7!)}{7!} = 10\cdot 9\cdot 8 = 10\cdot 72 = 720.$$

2. (6 pts) Consider the sample space $S = \{o_1, o_2, o_3, o_4\}$. Suppose $\Pr(o_1) = 0.33$ and $\Pr(o_2) = 0.25$. If o_3 and o_4 are equally likely, complete the probability assignment for this probability space.

We know $\Pr(o_3) = \Pr(o_4)$ and that

$$\Pr(o_1) + \Pr(o_2) + \Pr(o_3) + \Pr(o_4) = 1$$
$$0.33 + 0.25 + 2\Pr(o_3) = 1$$
$$2\Pr(o_3) = 1 - 0.58$$
$$2\Pr(o_3) = 0.42$$
$$\Pr(o_3) = 0.21$$

$\Pr(o_1) = 0.33$ $\Pr(o_2) = 0.25$ $\Pr(o_3) =$ 0.21 $\Pr(o_4) =$ 0.21