

Hand-in Homework 2: wed, 2/4/15

Prop 27. If  $n \in \mathbb{Z}$ , then  $0 \cdot n = 0$ .

Proof. Let  $n \in \mathbb{Z}$  be arbitrary. Since 0 is the additive identity element of  $\mathbb{Z}$ ,  $0+0=0$ . Therefore,

$$(0+0) \cdot n = 0 \cdot n$$

$$0 \cdot n + 0 \cdot n = 0 \cdot n \quad (\text{right distributive law})$$

$$(0 \cdot n + 0 \cdot n) + (-0 \cdot n) = 0 \cdot n + (-0 \cdot n)$$

$$0 \cdot n + [0 \cdot n + (-0 \cdot n)] = 0 \cdot n + (-0 \cdot n) \quad (\text{associativity of multiplication})$$

$$0 \cdot n + 0 = 0 \quad (\text{additive inverses})$$

Hence,  $0 \cdot n = 0$ , as desired. ■

Prop. 29. If  $a \in \mathbb{Z}$ , then  $(-1) \cdot a = -a$ .

Proof. Let  $a \in \mathbb{Z}$  be arbitrary. Then

$$\begin{aligned} (-1) \cdot a + a &= (-1) \cdot a + 1 \cdot a \quad (1 \text{ is multiplicative} \\ &\quad \text{identity}) \\ &= (-1+1) \cdot a \quad (\text{distributive law}) \\ &= 0 \cdot a \quad (\text{additive inverses}) \\ &= 0 \quad (\text{Prop. 27}). \end{aligned}$$

By the uniqueness of additive inverses, (Fact 21b), we conclude that  $(-1) \cdot a = -a$ , as desired. ■

Cor. 30. It is true that  $(-1) \cdot (-1) = 1$ .

Proof. Applying Propositions 29 and 28, with  $a=-1$ , we obtain  $(-1) \cdot (-1) = -(-1) = 1$ , as desired. ■