Quiz 4: Thursday, June 22, 2006

Show your reasoning.

1. (10 points) Using complete sentences, state each of the following formally (precisely).
   (a) Associative Law of Addition in the set $\mathbb{N}$ of natural numbers
   \[
   \text{For all } a, b, c \in \mathbb{N}, \quad a + (b + c) = (a + b) + c.
   \]
   crucial quantifier!

   (b) Commutative Law of Multiplication in the set $\mathbb{N}$ of natural numbers
   \[
   \text{For all } a, b \in \mathbb{N}, \quad a \cdot b = b \cdot a.
   \]

2. (10 points) Let $A = \{0, 1, 2, 3, 4, 5, 6, 7\}$. Define the relation $F$ on $A$ by $a F b$ if and only if $a + b = 6$.
   (a) List the elements of $F$ (using set notation).
   \[F = \{(0,6), (1,5), (2,4), (3,3), (4,2), (5,1), (6,0)\} \text{.}\]

   (b) Is $F$ transitive? NO.

   Prove your answer. The relation $F$ is not transitive since, for example, 0+6 = 6 and 6+0 = 6, so (0,6),(6,0) $\in F$, but (0,0) $\notin F$ as 0+0 $\neq$ 6.