Quiz 6: Friday, June 30, 2006

Show your reasoning.

1. (5 points) Using complete sentences, state the Fundamental Theorem of Arithmetic.

   Every integer $n > 1$ is either a prime or a product of primes. Furthermore, the expression of $n$ as a product of primes is unique up to order.

2. (4 points) Find the canonical prime factorization of the integer $n = 1400$.

   $n = 1400$
   $= 14 \cdot 100$
   $= 2 \cdot 7 \cdot 2^2 \cdot 5^2$
   $= 2^3 \cdot 5^2 \cdot 7$

   Note: For the prime factorization to be considered “canonical,” the primes must be written in increasing order.

3. (6 points) Let $n = 3^{15} \cdot 11^{20} \cdot 37^2$. Determine whether each of the following integers divides $n$. Explain.

   (a) $3^{10} \cdot 11^{20} | n$ Since $10 \leq 15$, $20 \leq 20$ are the relevant exponents,

   $3^{10} | 3^{15}$ and $11^{20} | 11^{20}$, so

   $3^{10} \cdot 11^{20} | 3^{15} \cdot 11^{20} \cdot 37^2$.

   (b) $3^{20} \cdot 11^{20} \cdot 37$ No, since $20 > 15$, $3^{20} \nmid 3^{15}$ and so

   $3^{20} \cdot 11^{20} \cdot 37 \nmid 3^{15} \cdot 11^{20} \cdot 37^2$.

4. (5 points) Determine the number of positive divisors of the integer $n = 3,884,517,875 = 5^3 \cdot 7^5 \cdot 43^2$.

   $(3+1)(5+1)(2+1) = (4)(6)(3)$
   $= (24)(3)$
   $= 72$ positive divisors.