Consider the following polynomials.
\[ a(x) = 2x^2 + x - 1 \]
\[ b(x) = 4x^3 + 12x^2 - x - 3 \]

1. Use the Euclidean Algorithm to find \((a(x), b(x))\), the greatest common divisor of \(a(x)\) and \(b(x)\). Show your reasoning.

\[
\begin{align*}
2x^2 + x - 1 & \mid 4x^3 + 12x^2 - x - 3 \\
-4x^3 - 2x^2 & \quad \text{余 } -4x^2 + x - 1 \\
10x^2 + x - 3 & \quad \text{余 } -2x^2 + x \\
-10x^2 + 5x + 5 & \quad \text{余 } 2x - 1 \\
-4x + 2 & \quad \text{余 } 0
\end{align*}
\]

Thus,
\[ 4x^3 + 12x^2 - x - 3 = (2x^2 + x - 1)(2x^2 + x - 1) + (-4x + 2) \]
\[ 2x^2 + x - 1 = (-4x + 2)(-\frac{1}{2}x - \frac{1}{2}) + 0 \]

The last non-zero remainder is \(-4x + 2\), so the desired GCD is a monic polynomial which is a multiple of \(-4x + 2\).

So, \( \gcd(a(x), b(x)) = x - \frac{1}{2} \) (monic, \( x - \frac{1}{2} = -\frac{1}{4}(-4x + 2) \) is monic)

2. Use your work in the previous part to find polynomials \(f(x)\) and \(g(x)\) such that \((a(x), b(x)) = a(x)f(x) + b(x)g(x)\). Show your reasoning.

Solving equation 1 above for \(-4x + 2\), we obtain,
\[-4x + 2 = (4x^3 + 12x^2 - x - 3) - (2x^2 + x - 1)(2x^2 + x - 1) \]
\[-\frac{1}{4}(-4x + 2) = -\frac{1}{4}(4x^3 + 12x^2 - x - 3) + (\frac{1}{2}x + \frac{5}{4})(2x^2 + x - 1) \]
\[f(x) = \frac{1}{2}x + \frac{5}{4} \quad \quad g(x) = -\frac{1}{4}\]