MATH 34001 | Fundamental Concepts of Algebra | Spring 2015
MWF 11:00 | | Dr. Kracht

Name: KEY   Quiz Score: ____ /20

Quiz 2: Friday, January 30, 2015

To receive full credit, show all work necessary to justify answers and all steps of solutions and derivations clearly, in logical sequence, using notation developed in class. Write proofs in complete sentences (with proper capitalization, punctuation, subject, verb, etc.). Partial credit will be given only for significant progress toward a solution.

1. (10 pts) Complete each of the following definitions.

   (a) A integer n is said to be ***even*** if $n = 2k$ for some integer k.

   (b) A set is said to be ***closed*** under a binary operation if the combination of any two elements of the set is itself an element of the set.

   (c) The number $\underline{1}$ is said to be the ***multiplicative identity element*** of $\mathbb{Z}$ since $1 \cdot a = a$ for all $a \in \mathbb{Z}$.

2. (10 pts) Prove the following proposition.

Proposition. *If x is an even integer and y is an odd integer, then the product xy is even.*

Proof. Let x be an even integer and y an odd integer. Then there are integers k and l such that $x = 2k$ and $y = 2l+1$. Thus,

$$\begin{aligned} xy &= (2k)(2l+1) \\ &= 2[k(2l+1)], \text{ by associativity of mult.} \end{aligned}$$

Now $k(2l+1)$ is an integer, by the closure of $\mathbb{Z}$ under multiplication. Therefore, xy is even, as desired. ∎

□