MATH 34001
MWF 11:00

Fundamental Concepts of Algebra

Spring 2015
Dr. Kracht

Name: KEY Quiz Score: ____ /20

Quiz 3: Friday, February 6, 2015

To receive full credit, show all work necessary to justify answers and all steps of solutions and derivations clearly, in logical sequence, using notation developed in class. Write proofs in complete sentences (with proper capitalization, punctuation, subject, verb, etc.). Partial credit will be given only for significant progress toward a solution.

1. (6pts) Complete each of the following definitions.

   (a) For $a \in \mathbb{Z}$, the number $-a$ is said to be the ***additive inverse*** of a since $a + (-a) = 0$.

   (b) The number 1 is said to be the ***multiplicative identity element*** of $\mathbb{Z}$ since $1 \cdot a = a$ for <u>all</u> $a \in \mathbb{Z}$.

2. (7 pts) Prove the following proposition.

   Proposition. *If $n \in \mathbb{Z}$, then $0 \cdot n = 0$.*

   Proof. Let $n \in \mathbb{Z}$. Since 0 is the additive identity element, $0+0=0$. Therefore,

$$(0+0)\cdot n = 0 \cdot n \quad \text{(multiplying both sides by } n\text{)},$$
$$0\cdot n + 0 \cdot n = 0 \cdot n \quad \text{(distributive law)},$$
$$[0\cdot n + 0\cdot n] + (-(0\cdot n)) = 0\cdot n + (-(0\cdot n)), \quad \text{(adding } -(0\cdot n) \text{ to both sides)},$$
$$0\cdot n + [0\cdot n + (-(0\cdot n))] = 0\cdot n + (-(0\cdot n)) \quad \text{(associativity)},$$
$$0\cdot n + 0 = 0 \quad \text{(additive inverses)},$$
$$0 \cdot n = 0 \quad \text{(additive identity)}.$$

□

3. (7 pts) Explain why division by 0 is undefined.

   Division is defined to be multiplication by a multiplicative inverse, that is, $a \div b = a \cdot b^{-1}$. However, 0 has no multiplicative inverse. If it did, it would be the solution of the equation $0 \cdot \square = 1$. This has no solution, since $0 \cdot n = 0$ for all numbers n. (The proof to #2 above works for all real numbers, not just integers.)