MATH 34001 | Fundamental Concepts of Algebra | Spring 2015
MWF 11:00 | | Dr. Kracht

Name: KEY Quiz Score: /20

Quiz 7: Friday, March 13, 2015

1. (10 pts) Prove that addition in $\mathbb{C}$ is commutative. That is, if $z, w \in \mathbb{C}$, then $z + w = w + z$.

Proof. Suppose $z, w \in \mathbb{C}$, say $z = a+bi$ and $w = c+di$, where $a, b, c, d \in \mathbb{R}$. Then

$$\begin{aligned} z+w &= (a+bi)+(c+di) \\ &= (a+c)+(b+d)i && \text{, by def}^{\underline{n}} \text{ of addition in } \mathbb{C}, \\ &= (c+a)+(d+b)i && \text{, by commutativity of addition in } \mathbb{C}, \\ &= (c+di)+(a+bi) && \text{, by def}^{\underline{n}} \text{ of addition in } \mathbb{C}, \\ &= w+z, \end{aligned}$$

as desired. ∎

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2. (10 pts) Prove that if $z \in \mathbb{C}$ with $\bar{z} = z$, then $z \in \mathbb{R}$.

Proof. Let $z \in \mathbb{C}$, say $z = a+bi$, where $a, b \in \mathbb{R}$. Suppose $\bar{z} = z$. That is, $\overline{a+bi} = a+bi$. This implies $a - bi = a+bi$. By the definition of equivalence of complex numbers, this means that $-b = b$. But 0 is the only real number that is equal to its own additive inverse, so $b = 0$. Therefore, $z = a + 0i = a$, which is a real number, as desired. ∎