MATH 34001 | Fundamental Concepts of Algebra | Spring 2015
MWF 11:00 | | Dr. Kracht

Name: KEY Quiz Score: _____/20

Quiz 9: Friday, April 17, 2015

1. (4 pts) Complete the following definition.

   If $d, m \in \mathbb{Z}$ with $d \neq 0$, we write $d|m$ and say "d divides m" to mean

   there exists some $k \in \mathbb{Z}$ such that $m = dk$.

2. (6 pts) Let $d, m, n \in \mathbb{Z}$ with $d \neq 0$. Circle TRUE or FALSE for each of the following.

   (a) TRUE or FALSE: If $d \nmid m$ and $d \nmid n$, then $d|(m+n)$. e.g. $3 \nmid 1$ and $3 \nmid 4$ and also $3 \nmid (1+4)$

   (b) TRUE or FALSE: If $d \nmid m$ and $d \nmid n$, then $d \nmid (m+n)$. e.g. $3 \nmid 1$ and $3 \nmid 5$ but $3 | (1+5)$

   (c) TRUE or FALSE: If $d|m$ but $d \nmid n$, then $d \nmid (m+n)$.

3. (10 pts) Prove the following proposition.

   Proposition. *Let $d, m, n \in \mathbb{Z}$ with $d \neq 0$. If $d|m$ and $d|n$, then $d|(m+n)$.*

   Proof. Let $d, m, n \in \mathbb{Z}$ with $d \neq 0$. Suppose $d|m$ and $d|n$. Then there exist $k, l \in \mathbb{Z}$ such that

   $$m = dk \quad \text{and} \quad n = dl.$$

   We have

   $$\begin{aligned} m+n &= dk + dl \\ &= d(k+l) \quad \text{(distributive law)}. \end{aligned}$$

   Since $\mathbb{Z}$ is closed under addition, $k+l \in \mathbb{Z}$. Therefore, by definition of divides, $d|(m+n)$, as desired. ∎

□