Name: _____ Score: _____ /20

Differentiation Worksheet

1. Let x and $f(x)$ represent the given quantities. Fix $x = a$ and let h be a small positive number. Give an interpretation of the quantities

$$\frac{f(a+h) - f(a)}{h} \qquad \text{and} \qquad \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} \qquad \text{where}$$

   (a) x denotes time and $f(x)$ denotes the population of black squirrels at time x.

   (b) x denotes time (in hours) and $f(x)$ denotes distance from Kent (in miles).

   (c) x denotes altitude and $f(x)$ denotes atmospheric pressure.

   (d) x denotes the speed of a car (in mph) and $f(x)$ denotes the fuel economy of the car measured in miles per gallon (mpg).

2. Find an equation of the tangent line to the graph of the given function f at the specified point $x = a$. Use the following steps.

   (i) Find the y-coordinate, $f(a)$, of the point.

   (ii) Find the derivative, $f'(x)$, of the function. *You may use short-cuts rather than the definition of derivative here.*

   (iii) Find the slope, $f'(a)$, of the desired tangent line.

   (iv) Find an equation of the desired tangent line using the point-slope form of the equation of a line, $y - y_1 = m(x - x_1)$ where $x_1 = a$, $y_1 = f(a)$, and $m = f'(a)$.

   (v) Rewrite the equation of the tangent line in the form $y = mx + b$.

   Learn the steps—they might not be enumerated on the exam. Use exact values (rather than decimal approximations from a calculator) for all numbers.

   (a) $f(x) = \sqrt{x} - \dfrac{1}{\sqrt{x}}$, $x = 9$

   (b) $f(x) = x^3 - 6x^2 + 3x - 10$, $x = -2$

   (c) $f(x) = \left(x^3 - 1\right)\left(x^2 + x + 1\right)$, $x = -1$

   (d) $f(x) = \dfrac{x}{x^2 + 1}$, $x = 2$

3. Suppose that f and g are differentiable functions such that

$$f(2) = 3 \qquad f'(2) = -1 \qquad g(2) = -5 \qquad \text{and} \qquad g'(2) = 10.$$

   For each of the following, find an expression for $h'(x)$. Then evaluate $h'(2)$. *Write out all steps.*

   (a) $h(x) = 3f(x) - g(x)$

   (b) $h(x) = f(x)g(x)$

   (c) $h(x) = \dfrac{f(x)}{g(x)}$

   (d) $h(x) = \left(x^5 - 1\right)g(x)$

   (e) $h(x) = \dfrac{x + f(x)}{f(x) - g(x)}$

4. Let f be a differentiable function. Write a mathematical expression representing each of the following quantities.

   (a) The output of the function f when the input is 5.

   (b) The y-coordinate of the point on the graph $y = f(x)$ whose x-coordinate is 5.

   (c) The slope of the secant line to the graph $y = f(x)$ through the points where $x = 4$ and $x = 5$.

   (d) The slope of the line tangent to the graph $y = f(x)$ at the point where $x = 5$.

   (e) The average rate of change of f with respect to x between $x = 4$ and $x = 5$.

   (f) The instantaneous rate of change of f with respect to x at the point where $x = 5$.