Differentiation Worksheet

1. Let $x$ and $f(x)$ represent the given quantities. Fix $x = a$ and let $h$ be a small positive number. Give an interpretation of the quantities

$$
\frac{f(a + h) - f(a)}{h} \quad \text{and} \quad \lim_{h \to 0} \frac{f(a + h) - f(a)}{h} \quad \text{where}
$$

(a) $x$ denotes time and $f(x)$ denotes the population of black squirrels at time $x$.
(b) $x$ denotes time (in hours) and $f(x)$ denotes distance from Kent (in miles).
(c) $x$ denotes altitude and $f(x)$ denotes atmospheric pressure.
(d) $x$ denotes the speed of a car (in mph) and $f(x)$ denotes the fuel economy of the car measured in miles per gallon (mpg).

2. Find an equation of the tangent line to the graph of the given function $f$ at the specified point $x = a$. Use the following steps.

(i) Find the $y$-coordinate, $f(a)$, of the point.
(ii) Find the derivative, $f'(x)$, of the function. You may use short-cuts rather than the definition of derivative here.
(iii) Find the slope, $f'(a)$, of the desired tangent line.
(iv) Find an equation of the desired tangent line using the point-slope form of the equation of a line, $y - y_1 = m(x - x_1)$ where $x_1 = a$, $y_1 = f(a)$, and $m = f'(a)$.
(v) Rewrite the equation of the tangent line in the form $y = mx + b$.

Learn the steps—they might not be enumerated on the exam. Use exact values (rather than decimal approximations from a calculator) for all numbers.

(a) $f(x) = \sqrt{x} - \frac{1}{\sqrt{x}}$, $x = 9$
(b) $f(x) = x^3 - 6x^2 + 3x - 10$, $x = -2$
(c) $f(x) = (x^3 - 1)(x^2 + x + 1)$, $x = -1$
(d) $f(x) = \frac{x}{x^2 + 1}$, $x = 2$

3. Suppose that $f$ and $g$ are differentiable functions such that

$$f(2) = 3 \quad f'(2) = -1 \quad g(2) = -5 \quad \text{and} \quad g'(2) = 10.$$

For each of the following, find an expression for $h'(x)$. Then evaluate $h'(2)$. Write out all steps.

(a) $h(x) = 3f(x) - g(x)$
(b) $h(x) = f(x)g(x)$
(c) $h(x) = \frac{f(x)}{g(x)}$
(d) $h(x) = (x^5 - 1)g(x)$
(e) $h(x) = \frac{x + f(x)}{f(x) - g(x)}$

4. Let $f$ be a differentiable function. Write a mathematical expression representing each of the following quantities.

(a) The output of the function $f$ when the input is 5.
(b) The $y$-coordinate of the point on the graph $y = f(x)$ whose $x$-coordinate is 5.
(c) The slope of the secant line to the graph $y = f(x)$ through the points where $x = 4$ and $x = 5$.
(d) The slope of the line tangent to the graph $y = f(x)$ at the point where $x = 5$.
(e) The average rate of change of $f$ with respect to $x$ between $x = 4$ and $x = 5$.
(f) The instantaneous rate of change of $f$ with respect to $x$ at the point where $x = 5$. 