Differentiation Hints

Sometimes simplifying a function *before* differentiating can make the problem much easier. This will also help you to avoid making silly errors.

Example 1. Let $f(x) = \dfrac{1}{x^2 \sqrt[3]{x}}$. Find $f'(x)$.

Long, Nasty Way. Here we use the Quotient and Product Rules.

$$f'(x) = \frac{0 \cdot (x^2 \sqrt[3]{x}) - 1 \cdot \left[x^2 \left(\frac{1}{3}x^{-2/3}\right) + 2x\sqrt[3]{x}\right]}{(x^2 \sqrt[3]{x})^2}.$$

Short, Easy Way. Here we use only the Power Rule.

$$\text{Write} \quad f(x) = \frac{1}{x^2 \sqrt[3]{x}} = x^{-7/3}.$$

$$\text{Thus,} \quad f'(x) = -\tfrac{7}{3}x^{-10/3}.$$

Example 2. Let $f(x) = \ln\left(\dfrac{4x^2}{e^x}\right)$. Find $f'(x)$.

Long, Very Nasty Way. Here we use the Chain Rule and the Quotient Rule.

$$f'(x) = \frac{1}{\frac{4x^2}{e^x}} \left\{ \frac{8x\, e^x - 4x^2\, e^x}{(e^x)^2} \right\}.$$

Short, Easy Way. Use the Laws of Logarithms first.

$$\begin{aligned} \text{Write} \quad f(x) &= \ln\left(\frac{4x^2}{e^x}\right) \\ &= \ln 4 + \ln\left(x^2\right) - \ln(e^x) \\ &= \ln 4 + 2\ln x - x. \\ \text{Thus,} \quad f'(x) &= 0 + 2 \cdot \frac{1}{x} - 1 \\ &= \frac{2}{x} - 1. \end{aligned}$$

Good Exercise. Practice your algebra by showing that, for the examples above, the function obtained by differentiating f the long, nasty way is actually the same as the function obtained by differentiating f the short, easy way.