

Notes on Notation in Calculus

Notation in calculus can be confusing. This note will address a few common mistakes. Please see your instructor for clarification, if necessary.

1. Use grouping symbols where necessary. You may use parentheses (), brackets [], or braces { } to indicate grouping.

(a) Be careful to distinguish between *subtraction* and *multiplication by a negative quantity*. For example,

$$15 - 2 \cdot 4 = 7;$$

$$\text{but } 15(-2) \cdot 4 = -120.$$

(b) A dot \cdot (even a really big dot \bullet) can NOT replace grouping symbols. For example,

$$3(x + 1) = 3x + 3;$$

$$\text{but } 3 \cdot x + 1 = 3x + 1;$$

$$\text{and even } 3 \bullet x + 1 = 3x + 1.$$

This is because multiplication has higher precedence than addition. Thus, in the last two computations, the multiplication, $3 \times x$, was performed before the addition. To force the addition, $x + 1$, to be performed before the multiplication (in other words, to multiply the entire *sum*, $x + 1$, by 3), we must enclose the sum, $x + 1$, in grouping symbols.

2. The equals sign ($=$) is always used to tie together two mathematical expressions which are equal to one another. (That is, expressions which have *exactly the same value*.)

(a) In particular, be careful not to put equals signs between a function and its derivative. These are almost never equal to one another. (In fact, the only functions for which $f = f'$ are constant multiples of the natural exponential function.)

(b) However, you *should* use equals signs between steps when you are simplifying an expression. This is because you are *not* changing the value of the expression. Your goal, in fact, is to produce a simpler expression which has *exactly the same value* as the original expression. Also remember to write out the whole expression at each step. For example,

$$\begin{aligned} \frac{d}{dx} \left(\frac{-2}{3\sqrt{x}} \right) &= \frac{d}{dx} \left(\frac{-2}{3} \cdot \frac{1}{x^{1/2}} \right) \\ &= -\frac{2}{3} \cdot \frac{d}{dx} \left(x^{-1/2} \right) \\ &= -\frac{2}{3} \left(-\frac{1}{2} x^{-3/2} \right) \\ &= \frac{1}{3} x^{-3/2} \\ &= \frac{1}{3x\sqrt{x}}. \end{aligned}$$

It is helpful to the reader of your solution to stack the equals signs vertically as above.

3. Since the symbol " $\lim_{h \rightarrow 0}$ " is read "*the limit as h approaches zero OF . . .*," it cannot stand alone. That is, it must be followed *immediately* by the name of a function or an expression (but NOT an equals sign).

4. Similarly, the symbol " $\frac{d}{dx}$ " is read "*the derivative with respect to x OF . . .*" and so it cannot stand alone. That is, it must be followed *immediately* by the name of a function or an expression (but NOT an equals sign).