Name: _____ Score: /20

Review Assignment II

due Friday, April 29, 2005

Show all your work.

For each problem completed (with all work), you will receive 1 point (for a possible 10 points). I will randomly choose one problem to grade for a possible 10 additional points.

1. A rocket is traveling with velocity $v(t) = 3t^2 + t$ feet per second at time t seconds after take-off. *Include appropriate units with your answers.*

   (a) Find the acceleration $a(t)$ of the rocket after t seconds.

   (b) Find the acceleration of the rocket after 10 seconds.

   (c) If the rocket took off from a platform 12 feet above the ground, find the height $h(t)$ of the rocket above the ground at t seconds.

   (d) Find the height of the rocket after 10 seconds.

2. Suppose the function f has the following properties.

$f(0) = 5$ $\quad f'(-4) = 0$ $\quad f''(-2) = 0$

$f'(0) = 0$ $\quad f''(2) = 0$

$f'(4) = 0$ $\quad f''(4) = 0$

(a) Complete the sign chart for f' with the phrases *"increasing"*, *"decreasing"*, *"concave up"*, or *"concave down"*.

interval	$(-\infty, -4)$	$(-4, 0)$	$(0, 4)$	$(4, \infty)$
$f'(x)$	$-$	$+$	$-$	$-$
f is				

(b) Complete the sign chart for f'' with the phrases *"increasing"*, *"decreasing"*, *"concave up"*, or *"concave down"*.

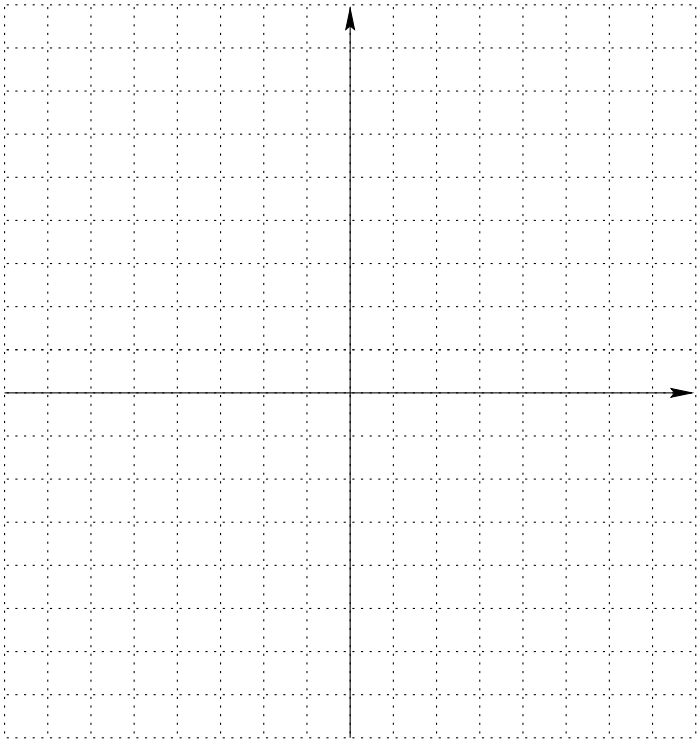
interval	$(-\infty, -2)$	$(-2, 2)$	$(2, 4)$	$(4, \infty)$
$f''(x)$	$+$	$-$	$+$	$-$
f is				

(c) Give the x-coordinates for all of the following. (Write "none" if there aren't any.)

critical points of f: $x =$ _____ relative maximum points of f: $x =$ _____

inflection points of f: $x =$ _____ relative minimum points of f: $x =$ _____

(d) Sketch a possible graph of the function f. Indicate all critical points and inflection points as well as other important behavior very clearly. (You are not given the y-coordinates of most points. Label these points as $(2, f(2))$, for example.)



3. Let $f(x) = \dfrac{1}{(x+2)(x-100)}$. Then $f'(x) = \dfrac{-2(x-49)}{(x+2)^2(x-100)^2}$.

Answer each of the following. *You must show all your steps carefully so that I know you are using calculus rather than relying on your grapher. When performing a computation, start with the mathematical symbol for the quantity you are computing. When solving an equation, write the complete equation at each step.*

(a) Give the ordered pairs for all x- and y-intercepts of f.

(b) Give the equations of all vertical asymptotes of f.

(c) Find the x-coordinates of all critical points of f.

(d) Construct a sign chart for f' indicating the sign of f' and the corresponding behavior of f. *Justify your answer.*

(e) Give the x-coordinates of all relative maximum points of f. (Write "none" if there aren't any.)

(f) Give the x-coordinates of all relative minimum points of f. (Write "none" if there aren't any.)

4. Let $f(x) = \sqrt[5]{x+1} - 2$. Find $f'(x) =$ and $f''(x) =$.

(a) Find the domain of f.

(b) Give the ordered pairs for all y-intercepts of f.

(c) Find the x-coordinates of all critical points of f.

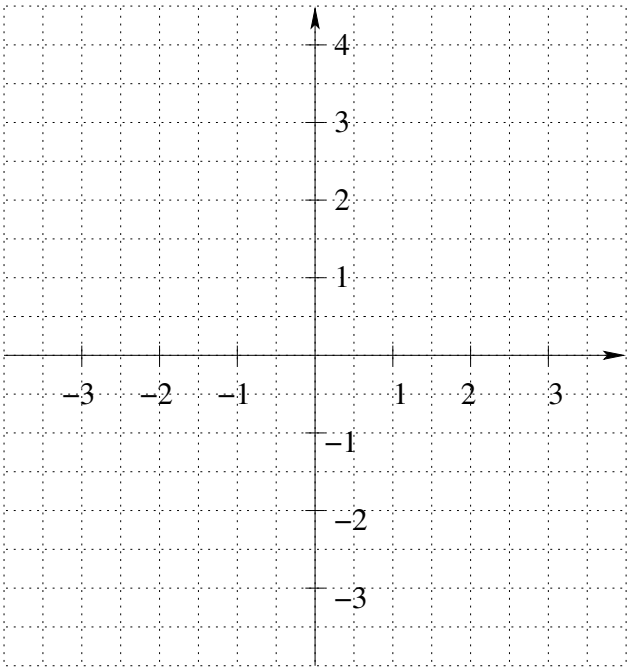
(d) Find the y-coordinates of all critical points of f.

(e) Construct a sign chart for f' indicating the sign of f' and the corresponding behavior of f. *Justify your answer.*

(f) Find the x-coordinates of all possible inflection points of f.

(g) Construct a sign chart for f'' indicating the sign of f'' and the corresponding behavior of f. *Justify your answer.*

(h) Sketch the graph of the function f. Indicate all intercepts, critical points, and inflection points as well as other important behavior very clearly.



5. Let $f(x) = 3e^{-2x^2}$. Find $f'(x) =$ and $f''(x) =$.

(a) Find the domain of f.

(b) Give the ordered pairs for all y-intercepts of f.

(c) Find the x-coordinates of all critical points of f.

(d) Find the y-coordinates of all critical points of f.

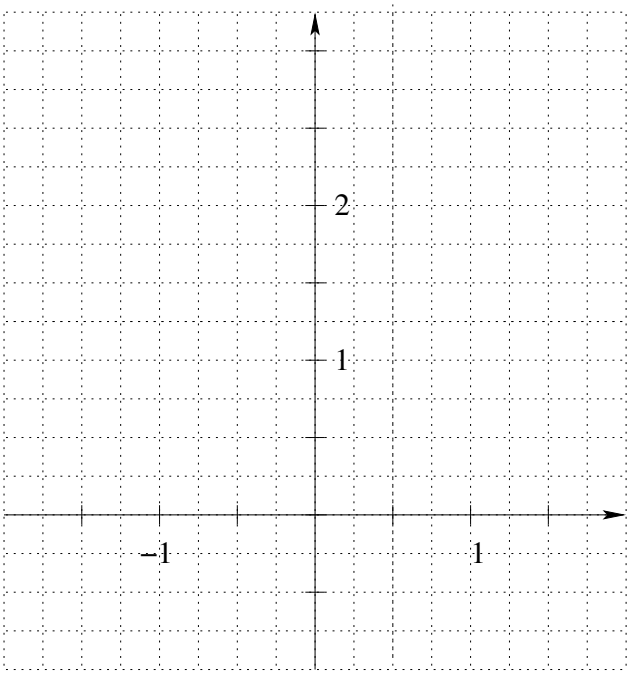
(e) Construct a sign chart for f' indicating the sign of f' and the corresponding behavior of f. *Justify your answer.*

(f) Find the x-coordinates of all possible inflection points of f.

(g) Find the y-coordinates of all possible inflection points of f.

(h) Construct a sign chart for f'' indicating the sign of f'' and the corresponding behavior of f. *Justify your answer.*

(i) Sketch the graph of the function f. Indicate all intercepts, critical points, and inflection points as well as other important behavior very clearly.
Hint: The line $y = 0$ is an asymptote.



6. A store can sell 20 HDTV sets per week at a price of \$400 each. The manager estimates that for each \$10 price reduction, she will sell 2 more HDTV's each week. The HDTV's cost the store \$200 each. Let x represent the number of \$10 price reductions. *(All of the following questions refer to weekly sales of HDTV's.)*

   (a) Express the price p of an HDTV as a function of x.

   (b) Express the quantity q of HDTV's sold weekly as a function of x.

   (c) Express revenue R as a function of x.

   (d) Express cost C as a function of x.

   (e) Express profit P as a function of x.

   (f) Find $P'(x)$.

   (g) Use calculus to find the value of x for which the store's profits are **maximal**. *Show your reasoning carefully. Verify that you have indeed found the absolute maximum point of the function.*

   (h) What price should the store set to maximize profits?

   (i) What quantity will the store sell at this price?

   (j) What is the maximal profit?

7. Using calculus, show that of all rectangles whose area is 1000 ft^2, the one with minimal perimeter is a square. *Show your reasoning. Be sure to:*

   - *Introduce all variables with "Let" statements. Include the units.*
   - *Draw and label a diagram.*
   - *Verify that you have indeed found the maximum or minimum point (on the appropriate domain).*
   - *Answer the question posed in the problem in a complete sentence, using appropriate units.*

8. An automobile dealer expects to sell 400 cars a year. The cars cost $11,000 each plus a fixed charge of $500 per delivery. It costs $1000 to store a car for a year. Find the order size and the number of orders per year that minimize inventory costs. *Show your reasoning. Be sure to:*

    - *Introduce all variables with "Let" statements. Include the units.*
    - *Verify that you have indeed found the maximum or minimum point (on the appropriate domain).*
    - *Answer the question posed in the problem in a complete sentence, using appropriate units.*

9. The reproduction function for the Canadian snowshoe hare is estimated to be $f(p) = -0.025p^2 + 4p$, where p and $f(p)$ are in thousands. Find the population that gives the maximum sustainable yield, and the size of the yield. *Show your reasoning. Be sure to:*

    - *Introduce all variables with "Let" statements. Include the units.*
    - *Verify that you have indeed found the maximum or minimum point (on the appropriate domain).*
    - *Answer the question posed in the problem in a complete sentence, using appropriate units.*

10. *Do this without a calculator.*

Let $f(x) = 4^x$.

(a) Give the ordered pairs for all intercepts of f. *Indicate "none" if there aren't any. Show the algebra explicitly.*

i. y-intercepts:

ii. x-intercepts:

(b) Complete the table of values.

x	$f(x)$
-2	
-1	
$-1/2$	
0	
$1/2$	
1	
2	

(c) Evaluate each limit. *Additional work not needed here.*

i. $\lim_{x \to -\infty} 4^x =$

ii. $\lim_{x \to \infty} 4^x =$

(d) Give the equations of all asymptotes of f. *Indicate "none" if there aren't any. Additional work not needed here.*

i. vertical asymptotes:

ii. horizontal asymptotes:

(e) Sketch the graph $y = f(x)$. *Label all points computed above with their ordered pairs and all asymptotes with their equations.*

(f) Sketch the graph $y = \log_4 x$ on the same coordinate system. *Label at least six points with their ordered pairs and all asymptotes with their equations.*