

Name: _____ Score: _____ /100
 (110 pts available)

EXAM 1: Version A

Show your reasoning for full credit.

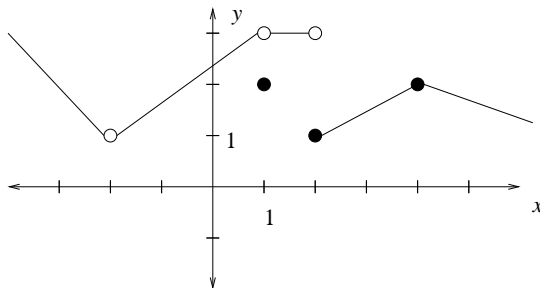
Some Formulas

Product Rule: If $p(x) = f(x) \cdot g(x)$, then $p'(x) = f'(x) \cdot g(x) + f(x) \cdot g'(x)$.

Quotient Rule: If $q(x) = \frac{f(x)}{g(x)}$, then $q'(x) = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{[g(x)]^2}$.

Generalized Power Rule: If $h(x) = [g(x)]^n$, then $h'(x) = n[g(x)]^{n-1} \cdot g'(x)$.

1. (10 points) The function f is depicted below.



(a) Find each of the following or state "does not exist" ("dne").

$\lim_{x \rightarrow -2^-} f(x) = \underline{\hspace{2cm}}$	$\lim_{x \rightarrow 1^-} f(x) = \underline{\hspace{2cm}}$	$\lim_{x \rightarrow 2^-} f(x) = \underline{\hspace{2cm}}$	$\lim_{x \rightarrow 4^-} f(x) = \underline{\hspace{2cm}}$
$\lim_{x \rightarrow -2^+} f(x) = \underline{\hspace{2cm}}$	$\lim_{x \rightarrow 1^+} f(x) = \underline{\hspace{2cm}}$	$\lim_{x \rightarrow 2^+} f(x) = \underline{\hspace{2cm}}$	$\lim_{x \rightarrow 4^+} f(x) = \underline{\hspace{2cm}}$
$\lim_{x \rightarrow -2} f(x) = \underline{\hspace{2cm}}$	$\lim_{x \rightarrow 1} f(x) = \underline{\hspace{2cm}}$	$\lim_{x \rightarrow 2} f(x) = \underline{\hspace{2cm}}$	$\lim_{x \rightarrow 4} f(x) = \underline{\hspace{2cm}}$
$f(-2) = \underline{\hspace{2cm}}$	$f(1) = \underline{\hspace{2cm}}$	$f(2) = \underline{\hspace{2cm}}$	$f(4) = \underline{\hspace{2cm}}$

(b) Circle YES or NO for each of the following.

i. Is f continuous at -2 ? YES NO

ii. Is f continuous at 1 ? YES NO

iii. Is f continuous at 2 ? YES NO

iv. Is f continuous at 4 ? YES NO

Multiple Choice: 10 questions at 3 points each.

Circle the letter of the best response

2. Evaluate $\lim_{x \rightarrow 2} (3x^2 + 5)$.
- (a) 0
(b) 11
(c) 17
(d) 41
(e) none of these
3. Evaluate $\lim_{x \rightarrow 6} \frac{x - 6}{x^2 - 36}$.
- (a) $\frac{1}{12}$
(b) 12
(c) 0
(d) does not exist
(e) none of these
4. Let $f(x) = \begin{cases} x & \text{if } x \leq 1; \\ 2x & \text{if } x > 1. \end{cases}$
Is $f(x)$ continuous at $x = 1$?
- (a) Yes
(b) No, since $f(1)$ is undefined.
(c) No, since $f(x)$ is a piecewise-defined function
(d) No, since $\lim_{x \rightarrow 1} f(x)$ does not exist.
(e) none of these
5. Let $f(x) = x^3 + 2x + 1$. What is the average rate of change in $f(x)$ between the values $x = 1$ and $x = 5$?
- (a) 29 because $f'(3) = 29$
(b) 41 because $\frac{f'(1) + f'(5)}{2} = 41$
(c) 33 because $\frac{f(5) - f(1)}{4} = 33$
(d) 132 because $f(5) - f(1) = 132$
(e) none of these
6. Find the instantaneous rate of change of the function $f(x) = 10x^4 + 5$ at $x = -2$.
- (a) -80
(b) -320
(c) 320
(d) -315
(e) none of these
7. Find the slope of the tangent line to $f(x) = \frac{1}{x}$ at $x = 3$.
- (a) -1
(b) $-\frac{1}{3}$
(c) -3
(d) $-\frac{1}{9}$
(e) none of these
8. Find the derivative of $g(x) = 5x^{100}$.
- (a) $g'(x) = 5x^{99}$
(b) $g'(x) = 500x^{99}$
(c) $g'(x) = 500x$
(d) $g'(x) = 500x^{101}$
(e) none of these
9. Use the product rule to find the derivative of $h(t) = (7t - 3)(2t + 5)$. *The answer need not be simplified.*
- (a) $h'(t) = 7t(2t + 5) + 2t(7t - 3)$
(b) $h'(t) = 7(2t + 5) + 2(7t - 3)$
(c) $h'(t) = 14t^2$
(d) $h'(t) = 14$
(e) none of these

10. Suppose a company's revenue function is $R(x) = 25x + 4\sqrt{x}$, in dollars, where x is the number of widgets produced. Find and interpret $R'(100)$.

- (a) The company's revenue from the 100th widget is \$25.40.
- (b) The company's total revenue from the first 100 widgets is \$25.20.
- (c) The company's total revenue from the first 100 widgets is \$2540.
- (d) The company's average revenue from the first 100 widgets is \$25.40 per widget.
- (e) none of these

11. Suppose a company's revenue function is $R(x) = 25x + 4\sqrt{x}$, in dollars, where x is the number of widgets produced. Find and interpret $R'(100)$.

- (a) The company's revenue from the 100th widget is approximately \$25.20.
- (b) The company's total revenue from the first 100 widgets is \$25.40.
- (c) The company's average revenue from the first 100 widgets is \$25.40 per widget.
- (d) The company's average revenue from the first 100 widgets is \$25.20 per widget.
- (e) none of these

Long Answer

Write all work carefully and neatly for full credit.

12. (15 pts) Find each derivative, simplifying your answer. (*You may use short-cuts.*)

(a) Find $\frac{d}{dz} \sqrt{9z^2 - 25}$.

(c) Find $g'(x)$ where $g(x) = \frac{x^2 + 3}{x^2 - 3}$

(b) Find $\frac{dy}{dx}$ where $y = x^2(3x - 1)^5$

13. (10 pts) A rocket rises to a height $h(t) = 1.5t^2 + 4t$ feet in t seconds. *Include units with each of the following.*

- (a) Find the height of the rocket after 10 seconds.
 - (b) Find the velocity of the rocket after t seconds.
 - (c) Find the velocity of the rocket after 10 seconds.
 - (d) Find the acceleration of the rocket after t seconds.
 - (e) Find the acceleration of the rocket after 10 seconds.
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14. (5 pts) Evaluate $\frac{d^3}{dx^3} \sqrt[3]{x^5}$.

15. (10 points) Find an equation of the line tangent to the graph of the function $f(x) = x^3 + 4x^2 - 5$ at the point where $x = -1$ by following the given steps. *Show your reasoning.*

- (a) Find the y -coordinate of the point on the curve $y = f(x)$ where $x = -1$.
- (b) Find the derivative $f'(x)$. *You may use short-cuts.*
- (c) Find the slope of the line tangent to the curve $y = f(x)$ at the point where $x = -1$.
- (d) Find the equation of the line tangent to the curve $y = f(x)$ at the point where $x = -1$. *Write it in the form "y = mx + b."*

16. (15 points) Use the **definition of derivative** to find and simplify the derivative, $f'(x)$, for the function $f(x) = 6 - 2x^2$.

Bonus. (10 points) Use the **definition of derivative** to prove that if f and g are differentiable functions, then

$$(f - g)'(x) = f'(x) - g'(x)$$