MATH 11012 | Intuitive Calculus | Spring 2011

Circle one: 8:50 5:30 | Ms. Kracht

Name: _____ Score: _____/100

. *(110 pts available)*

EXAM 1: Version B

Show your reasoning for full credit.

Some Formulas

Product Rule: If $p(x) = f(x) \cdot g(x)$, then $p'(x) = f'(x) \cdot g(x) + f(x) \cdot g'(x)$.

Quotient Rule: If $q(x) = \dfrac{f(x)}{g(x)}$, then $q'(x) = \dfrac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{[g(x)]^2}$.

Generalized Power Rule: If $h(x) = [g(x)]^n$, then $h'(x) = n[g(x)]^{n-1} \cdot g'(x)$.

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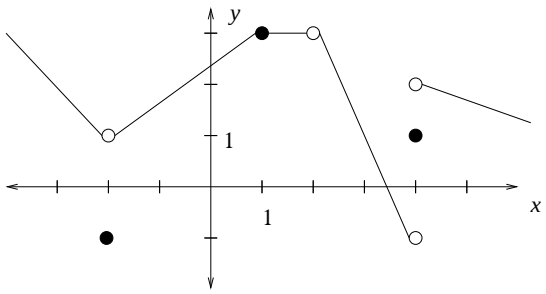
1. (10 points) The function f is depicted below.



(a) Find each of the following or state "does not exist" ("dne").

$\lim_{x\to -2^-} f(x) =$ _____	$\lim_{x\to 1^-} f(x) =$ _____	$\lim_{x\to 2^-} f(x) =$ _____	$\lim_{x\to 4^-} f(x) =$ _____
$\lim_{x\to -2^+} f(x) =$ _____	$\lim_{x\to 1^+} f(x) =$ _____	$\lim_{x\to 2^+} f(x) =$ _____	$\lim_{x\to 4^+} f(x) =$ _____
$\lim_{x\to -2} f(x) =$ _____	$\lim_{x\to 1} f(x) =$ _____	$\lim_{x\to 2} f(x) =$ _____	$\lim_{x\to 4} f(x) =$ _____
$f(-2) =$ _____	$f(1) =$ _____	$f(2) =$ _____	$f(4) =$ _____

(b) Circle YES or NO for each of the following.

i. Is f continuous at -2? YES NO

ii. Is f continuous at 1? YES NO

iii. Is f continuous at 2? YES NO

iv. Is f continuous at 4? YES NO

Multiple Choice: 10 questions at 3 points each.

Circle the letter of the best response

2. Evaluate $\lim_{x\to 2}\left(3x^2+5\right)$.

   (a) 11
   (b) 17
   (c) 41
   (d) 0
   (e) none of these

3. Evaluate $\lim_{x\to 6}\dfrac{x-6}{x^2-36}$.

   (a) 0
   (b) does not exist
   (c) $\frac{1}{12}$
   (d) 12
   (e) none of these

4. Let $f(x)=\begin{cases} x & \text{if } x\le 1;\\ 2x & \text{if } x>1.\end{cases}$
   Is $f(x)$ continuous at $x=1$?

   (a) Yes
   (b) No, since $f(x)$ is a piecewise-defined function
   (c) No, since $\lim_{x\to 1} f(x)$ does not exist.
   (d) No, since $f(1)$ is undefined.
   (e) none of these

5. Let $f(x)=x^3+2x+1$. What is the average rate of change in $f(x)$ between the values $x=1$ and $x=5$?

   (a) 41 because $\dfrac{f'(1)+f'(5)}{2}=41$
   (b) 33 because $\dfrac{f(5)-f(1)}{4}=33$
   (c) 132 because $f(5)-f(1)=132$
   (d) 29 because $f'(3)=29$
   (e) none of these

6. Find the instantaneous rate of change of the function $f(x)=10x^4+5$ at $x=-2$.

   (a) -320
   (b) 320
   (c) -315
   (d) -80
   (e) none of these

7. Find the slope of the tangent line to $f(x)=\dfrac{1}{x}$ at $x=3$.

   (a) $-\frac{1}{3}$
   (b) -3
   (c) $-\frac{1}{9}$
   (d) -1
   (e) none of these

8. Find the derivative of $g(x)=5x^{100}$.

   (a) $g'(x)=500x^{99}$
   (b) $g'(x)=500x$
   (c) $g'(x)=500x^{101}$
   (d) $g'(x)=5x^{99}$
   (e) none of these

9. Use the product rule to find the derivative of $h(t)=(7t-3)(2t+5)$. *The answer need not be simplified.*

   (a) $h'(t)=7(2t+5)+2(7t-3)$
   (b) $h'(t)=14t^2$
   (c) $h'(t)=14$
   (d) $h'(t)=7t(2t+5)+2t(7t-3)$
   (e) none of these

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10. Suppose a company's revenue function is $R(x) = 25x + 4\sqrt{x}$, in dollars, where x is the number of widgets produced. Find and interpret $R(100)$.

    (a) The company's total revenue from the first 100 widgets is \$25.20.

    (b) The company's total revenue from the first 100 widgets is \$2540.

    (c) The company's average revenue from the first 100 widgets is \$25.40 per widget.

    (d) The company's revenue from the 100th widget is \$25.40.

    (e) none of these

11. Suppose a company's revenue function is $R(x) = 25x + 4\sqrt{x}$, in dollars, where x is the number of widgets produced. Find and interpret $R'(100)$.

    (a) The company's total revenue from the first 100 widgets is \$25.40.

    (b) The company's average revenue from the first 100 widgets is \$25.40 per widget.

    (c) The company's average revenue from the first 100 widgets is \$25.20 per widget.

    (d) The company's revenue from the 100th widget is approximately \$25.20.

    (e) none of these

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Long Answer

Write all work carefully and neatly for full credit.

12. (15 pts) Find each derivative, simplifying your answer. *(You may use short-cuts.)*

    (a) Find $\dfrac{d}{dz}\sqrt{16z^2 - 49}$.

    (b) Find $\dfrac{dy}{dx}$ where $y = x^2(2x-1)^7$

    (c) Find $g'(x)$ where $g(x) = \dfrac{x^2 - 5}{x^2 + 8}$

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13. (10 pts) A rocket rises to a height $h(t) = 0.5t^2 + 8t$ feet in t seconds. *Include units with each of the following.*

    (a) Find the height of the rocket after 10 seconds.

    (b) Find the velocity of the rocket after t seconds.

    (c) Find the velocity of the rocket after 10 seconds.

    (d) Find the acceleration of the rocket after t seconds.

    (e) Find the acceleration of the rocket after 10 seconds.

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14. (5 pts) Evaluate $\dfrac{d^3}{dx^3}\sqrt[3]{x^5}$.

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15. (10 points) Find an equation of the line tangent to the graph of the function $f(x) = x^3 + 4x^2 - 5$ at the point where $x = -1$ by following the given steps. *Show your reasoning.*

    (a) Find the y-coordinate of the point on the curve $y = f(x)$ where $x = -1$.

    (b) Find the derivative $f'(x)$. *You may use short-cuts.*

    (c) Find the slope of the line tangent to the curve $y = f(x)$ at the point where $x = -1$.

    (d) Find the equation of the line tangent to the curve $y = f(x)$ at the point where $x = -1$. *Write it in the form "$y = mx + b$."*

16. (15 points) Use the **definition of derivative** to find and simplify the derivative, $f'(x)$, for the function $f(x) = 5-3x^2$.

Bonus. (10 points) Use the **definition of derivative** to prove that if f and g are differentiable functions, then

$$(f-g)'(x) = f'(x) - g'(x)$$

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