MATH 11012
Circle one: 8:50 5:30

Intuitive Calculus

Spring 2011
Ms. Kracht

Name: KEY

Score: /100
(113 pts available)

EXAM 3: Version A

NO CALCULATORS.

Multiple Choice: 17 questions at 4 points each.
Circle the letter of the best response.
Although your work will not be graded, you should write out complete and careful solutions to reduce the chance of error.

1. The lowest point of the graph of $f(x) = x^3 - 3x + 1$ on the interval $[-1, 3]$ is

C

(a) $(2, -9)$
(b) $(0, -3)$
(c) $(1, -1)$
(d) $(-1, 0)$
(e) none of these

$f'(x) = 3x^2 - 3$
$= 3(x^2 - 1)$
$= 3(x-1)(x+1)$

critical #'s:
$f'(x) = 0$
$3(x-1)(x+1) = 0$
$x - 1 = 0$ or $x + 1 = 0$
$x = 1$ or $x = -1$

Test endpts:
$f(-1)$ done
$f(3) = 3^3 - 3.3 + 1$
$= 27 - 9 + 1$
$= 19$ max

Test crit #'s:
$f(1) = 1^3 - 3.1 + 1 = 1 - 3 + 1 = -1$ min
$f(-1) = (-1)^3 - 3(-1) + 1 = -1 + 3 + 1 = 3$

2. The highest point of the graph of $f(x) = x^3 - 3x + 1$ on the interval $[-1, 3]$ is

A

(a) $(3, 19)$
(b) $(0, 1)$
(c) $(-1, 3)$
(d) $(3, 24)$
(e) none of these

work above ↑

3. True or False: Suppose f is continuous and that $f'(7) = 0$ and $f''(7) = -10$. Then we may conclude that f has an **absolute maximum** point at $x = 7$.

B

(a) True
(b) False

f has a relative minimum, but it need not be an absolute min, at $x = 7$.

12

4. True or False: If f is continuous on the interval $[1,5]$, then f must have an **absolute minimum value** on $[1,5]$.

A

(a) True   by the Extreme Value Theorem

(b) False

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5. True or False: If f is continuous on the interval $(1,5)$, then f must have an **absolute minimum value** on $(1,5)$.

B

(a) True

(b) False   interval not closed

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6. An automobile dealer can sell 8 cars per day at a price of $25,000. He estimates that for each $1000 price reduction, he can sell 3 more cars each day. Let x be the number of $1000 price reductions. Find an expression for the price per car (in dollars).

D

(a) $25,000x - 1000$

(b) $25,000 \times 1000x$

(c) $25,000 - 8x$

(d) $25,000 - 1000x$

(e) none of these

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7. An automobile dealer can sell 8 cars per day at a price of $25,000. He estimates that for each $1000 price reduction, he can sell 3 more cars each day. Let x be the number of $1000 price reductions. Find an expression for the number of cars sold per day.

A

(a) $8 + 3x$

(b) $8 - 1000x$

(c) $8x$

(d) $8 - 3x$

(e) none of these

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8. If the reproduction function for Albacore tuna is $f(p) = 5\sqrt[3]{p^2}$ (where both p and $f(p)$ are in thousands), find the sustainable yield for a population of 8000 tuna. (Evaluate $Y(8)$ where $Y(p)$ is the sustainable yield function.)

B

(a) 8,000 tuna

(b) 12,000 tuna

(c) 20,000 tuna

(d) 16,000 tuna

(e) none of these

$$Y(p) = f(p) - p = 5\sqrt[3]{p^2} - p$$
$$Y(8) = 5(\sqrt[3]{8})^2 - 8$$
$$= 5 \cdot 2^2 - 8$$
$$= 20 - 8 = 12 \quad \text{thousand tuna}$$

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9. Evaluate the integral: $\int 2.4x\,dx = 2.4\left(\frac{1}{2}x^2\right) + C = 1.2x^2 + C$

C

(a) $2.4x^2 + C$

(b) $4.8x^2 + C$

(c) $1.2x^2 + C$

(d) $2.4 + C$

(e) none of these

24

10. Evaluate the integral: $\int \frac{5}{x^4}\,dx$ $= \int 5x^{-4}dx = 5\left(-\frac{1}{3}x^{-3}\right)+C$

$= \frac{-5}{3x^3}+C$

(a) $\frac{1}{x^5}+C$

(b) $-\frac{15}{x^3}+C$

C

(c) $-\frac{5}{3x^3}+C$

(d) $5\ln|x^4|+C$

(e) none of these

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11. Evaluate the integral: $\int 8e^{2t}\,dt$ $= 8\left(\frac{1}{2}e^{2t}\right)+C$

$= 4e^{2t}+C$

B

(a) $16e^{2t}+C$

(b) $4e^{2t}+C$

(c) $16e^{3t}+C$

(d) $8e^{2t}+C$

(e) none of these

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12. Evaluate the integral: $\int \frac{6x^2-4x+7}{x^2}dx$ $= \int\left(\frac{6x^2}{x^2}-\frac{4x}{x^2}+\frac{7}{x^2}\right)dx$

A

(a) $6x-4\ln|x|-\frac{7}{x}+C$

$= \int\left(6-\frac{4}{x}+7x^{-2}\right)dx$

(b) $6x-4\ln|x|-\frac{7}{3x^3}+C$

$= 6x-4\ln|x|+7(-1x^{-1})+C$

(c) $6x-4\ln|x|-\frac{21}{x^3}+C$

$= 6x-4\ln|x|-\frac{7}{x}+C$

(d) $\frac{2x^3-2x^2+7x}{\frac{1}{3}x^3}+C$

(e) none of these

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13. Black squirrel flu is spreading on campus at a rate of $10e^{0.5t}$ new cases per day, where t is the number of days since the start of the epidemic. The epidemic began with 1 case. Find an expression for the total number of flu cases during the first t days.

B

(a) $20e^{0.5t}+1$

(b) $20e^{0.5t}-19$

(c) $10e^{0.5t}+1$

(d) $5e^{0.5t}$

(e) none of these

$f(t) = \int 10e^{0.5t}dt$

$= 10\left(\frac{1}{0.5}e^{0.5t}\right)+C$

$= 10(2)e^{0.5t}+C$

$= 20e^{0.5t}+C$

initial condition:

$f(0)=1$

$20e^{0.5(0)}+C=1$

$20+C=1$

$C=1-20$

$C=-19$

So $f(t)=20e^{0.5t}-19$

16

14. Evaluate the integral: $\int_e^{e^4} \frac{10}{x}\, dx.$ $= 10 \ln|x| \Big|_e^{e^4}$

$= 10(\ln e^4 - \ln e)$

$= 10(4-1)$

$= 10 \cdot 3$

$= 30$

C

(a) 0
(b) 3
(c) 30
(d) e^3
(e) none of these

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15. Find the area under the curve $y = x^2 + 4x$ from $x = 0$ to $x = 2$.

(a) 12
(b) $\frac{20}{3}$
(c) $\frac{32}{3}$
(d) $\frac{44}{3}$
(e) none of these

C

$A = \int_0^2 (x^2 + 4x)\, dx$

$= \left(\frac{1}{3}x^3 + 2x^2\right)\Big|_0^2$

$= \left[\frac{1}{3}(2)^3 + 2(2)^2\right] - \left[\frac{1}{3}\cdot 0^3 + 2 \cdot 0^2\right]$

$= \frac{8}{3} + 8 - 0$

$= \frac{8}{3} + \frac{24}{3} = \frac{32}{3}$

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16. A company is considering a new manufacturing process. The rate of savings from this process is expected to be $\frac{12,000}{t^2}$ dollars per year, where t is in years. Determine the total savings from the third through the sixth year.

(a) \$4000
(b) \$8000
(c) \$12,000
(d) \$2000
(e) none of these

D

$\int_3^6 \frac{12000}{t^2}\, dt = \int_3^6 12000 t^{-2}\, dt$

$= 12000(-t^{-1})\Big|_3^6$

$= -\frac{12000}{t}\Big|_3^6$

$= -\frac{12000}{6} - \left(-\frac{12,000}{3}\right)$ $= -2000 + 4000$

$= \$2000$

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17. Complete the statement of The Fundamental Theorem of Calculus.

Let f be continuous on $[a, b]$. Then $\int_a^b f(x)\, dx =$

(a) $f(b) - f(a)$
(b) $f'(b) - f'(a)$
(c) $F(b) - F(a)$, where F is any antiderivative of f.
(d) $F(a) - F(b)$, where F is any antiderivative of f.
(e) none of these

C

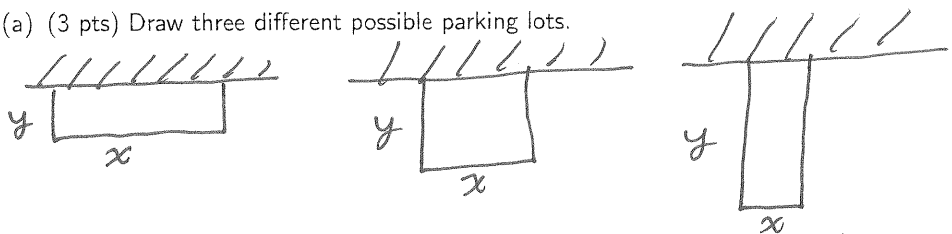
16

Long Answer

Write all work carefully and neatly for full credit.

18. (25 points) A company wants to build a rectangular parking lot along the side of a building using 800 feet of fence. The side along the building needs no fence. What are the dimensions of the largest such parking lot?

(a) (3 pts) Draw three different possible parking lots.



(b) (4 pts) Introduce your variables with "Let" statements. (Include the units.)

Let x be length of sides parallel to building, in feet.

Let y be length of sides perpendicular to building, in feet.

(c) (2 pts) Express the area A of the parking lot in terms of your variables.

$A = xy$

(d) (3 pts) Write a constraint equation relating your variables. *(Skip this step if you have only one variable.)*

$2y + x = 800$

$x = 800 - 2y$

(e) (2 pts) Express the area A in terms of one variable only.

$A = (800 - 2y)y = 800y - 2y^2$

(f) (5 pts) Use calculus to find the value of the variable for which A is maximal.

$A'(y) = 800 - 4y$

$A'(y) = 0$

$800 - 4y = 0$

$-4y = -800$

$y = 200$

(g) (4 pts) Verify that you have indeed found the **absolute maximum** point of A (on an appropriate interval, if necessary).

$A''(y) = -4$

$A''(200) = -4 < 0$

so relative max

Since 200 is the only critical #, A has its abs max @ $y = 200$.

when $y = 200$, $x = 800 - 2 \cdot 200 = 400$

(h) (2 pts) Answer the question in a complete sentence.

The largest parking lot is 200 by 400 ft with the long side parallel to the building.

19. (20 points) Evaluate each integral, simplifying your answer and writing out all work, including substitutions, if any. Be sure to use correct notation.

(a) $\int \sqrt{x^5 - 2}\, x^4 \, dx$

Let $u = x^5 - 2$

Then $\frac{du}{dx} = 5x^4$

So $du = 5x^4 dx$

Then $\frac{1}{5} du = x^4 dx$

$= \int \sqrt{u} \left(\frac{1}{5}\right) du$

$= \frac{1}{5} \int u^{1/2} du$

$= \frac{1}{5}\left(\frac{2}{3} u^{3/2}\right) + C$

$= \frac{2}{15} u^{3/2} + C$

$= \frac{2}{15} (x^5 - 2)^{3/2} + C$

Check:

$\frac{d}{dx}\left(\frac{2}{15}(x^5-2)^{3/2} + C\right)$

$= \frac{2}{15} \cdot \frac{3}{2} (x^5-2)^{1/2} (5x^4)$ Chain Rule

$= \frac{1}{5} (x^5-2)^{1/2} (5x^4)$

$= \sqrt{x^5-2}\; x^4$. ✓

(b) $\int_{-2}^{3} \frac{1}{4-x} dx$

Let $u = 4 - x$

Then $\frac{du}{dx} = -1$

$du = -1\, dx$

So $-1\, du = dx$

LL: If $x = -2$, $u = 4-(-2) = 6$

UL: If $x = 3$, $u = 4 - 3 = 1$.

$= \int_6^1 \frac{1}{u} (-1)\, du$

$= -\ln|u| \Big|_6^1$

$= -\ln|1| - (-\ln|6|)$

$= -\ln 1 + \ln 6$

$= -0 + \ln 6$

$= \underline{\ln 6}$.

$\boxed{20}$