Intuitive Calculus

Spring 2011 Ms. Kracht

Name: KE

Circle one: 8:50 5:30

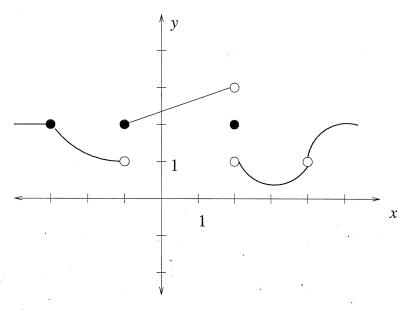
Quiz Score:

/25

Quiz 1: Version A

Show your reasoning. Use standard notation correctly.

1. (10 points) The function f is depicted below.



$$\lim_{x \to -3^{-}} f(x) = \underbrace{\frac{1}{2}}_{x \to -3^{-}} f(x) = \underbrace{\frac{1}{2}}_{x \to -1^{-}} f(x) = \underbrace{\frac{1}{2}}_{x \to -1^{-}} f(x) = \underbrace{\frac{1}{2}}_{x \to 2^{-}} f(x) = \underbrace{\frac{1}{2}}_{x \to 2^{-}} f(x) = \underbrace{\frac{1}{2}}_{x \to 4^{-}} f(x) = \underbrace{\frac{1}{2}$$

- (b) Circle YES or No for each of the following.
 - i. Is f continuous at -3? (YES) NO
 - ii. Is f continuous at -1? YES NO
 - iii. Is f continuous at 2? YES NO
 - iv. Is f continuous at 4? YES $\left(\begin{array}{c} \text{No} \end{array}\right)$

2. (5 points) Find the average rate of change of $f(x) = x^2 - 7x$ over the interval [1, 3].

$$\frac{f(3)-f(1)}{3-1} = \frac{3^2-7(3)}{2} - \frac{1^2-7\cdot1}{2}$$

$$= \frac{(9-21)-(1-7)}{2}$$

$$= -\frac{12-(-6)}{2} = -\frac{12+6}{2} = -\frac{6}{2} = -3$$

3. (10 points)

(a) State the definition of derivative.

$$f'(x) = \lim_{N \to 0} \frac{f(x+h) - f(x)}{h}$$

(b) Find the derivative f'(x) of the function $f(x) = 3x^2 - 10x + 5$ using the definition of derivative.

$$f(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{[3(x+h)^2 - 10(x+h) + 5] - [3x^2 - 10x + 5]}{h}$$

$$= \lim_{h \to 0} \frac{3(x^2 + 2hx + h^2) - 10x - 10h + 5 - 3x^2 + 30x - 5}{h}$$

$$= \lim_{h \to 0} \frac{3x^2 + 6hx + 3h^2 - 10h - 3x^2}{h}$$

$$= \lim_{h \to 0} \frac{(6hx + 2h^2 - 10h)}{h}$$

$$= \lim_{h \to 0} \frac{(6x + 3h - 10)}{h}$$

$$= \lim_{h \to 0} (6x + 3h - 10)$$

$$= 6x - 10$$

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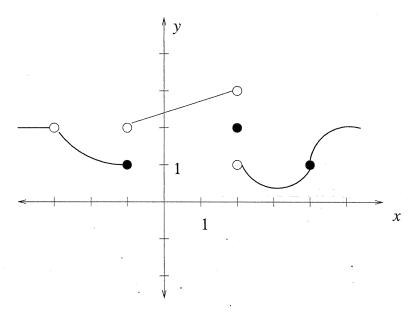
Quiz Score:

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Quiz 1: Version B

Show your reasoning. Use standard notation correctly.

1. (10 points) The function f is depicted below.



$$\lim_{x \to -3^{-}} f(x) = \frac{2}{2}$$

$$\lim_{x \to -3^{+}} f(x) = \frac{2}{2}$$

$$\lim_{x \to -3^{+}} f(x) = \frac{2}{2}$$

$$\lim_{x \to -1^{+}} f(x) = \frac{2}{2}$$

$$\lim_{x \to 2^{+}} f(x) = \frac{1}{2}$$

$$\lim_{x \to 4^{-}} f(x) = \frac{1}{2}$$

$$f(2) = \frac{2}{2}$$

- (b) Circle YES or No for each of the following
 - i. Is f continuous at -3? YES $\left(Nc \right)$
 - ii. Is f continuous at -1? YES $\left(N_{\mathcal{O}}\right)$
 - iii. Is f continuous at 2? YES $\left(\begin{array}{c} NO \end{array}\right)$
 - iv. Is f continuous at 4? $\left(YES\right)$ No

2. (5 points) Find the average rate of change of $f(x) = x^2 - 5x$ over the interval [1, 3].

$$\frac{f(3)-f(1)}{3-1} = \frac{(3^2-5\cdot3)-(1^2-5\cdot1)}{2}$$

$$= \frac{(9-15)-(1-5)}{2}$$

$$= -\frac{6-(-4)}{2} = -\frac{6+4}{2} = -\frac{7}{2} = -1.$$

3. (10 points)

(a) State the definition of derivative.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

(b) Find the derivative f'(x) of the function $f(x) = 2x^2 - 3x + 8$ using the definition of derivative.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{[2(x+h)^2 - 3(x+h) + 6] - [2x^2 - 3x + 8]}{h}$$

$$= \lim_{h \to 0} \frac{2(x^2 + 2hx + h^2) - 3x - 3h + 8 - 2x^2 + 3x - 8}{h}$$

$$= \lim_{h \to 0} \frac{2x^2 + 4hx + 2h^2 - 3h - 2x^2}{h}$$

$$= \lim_{h \to 0} \frac{Hhx + 2h^2 - 3h}{h}$$

$$= \lim_{h \to 0} \frac{h(4x + 2h - 3)}{h}$$

$$= \lim_{h \to 0} (4x + 2h - 3)$$

$$= 4x + 2 \cdot 0 - 3$$

$$= 4x - 3$$

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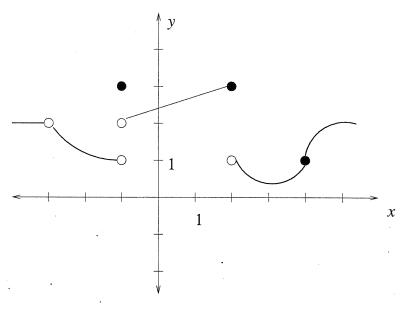
Quiz Score:

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Quiz 1: Version C

Show your reasoning. Use standard notation correctly.

1. (10 points) The function f is depicted below.



$$\lim_{x \to -3^{-}} f(x) = \frac{2}{2}$$

$$\lim_{x \to -3^{+}} f(x) = \frac{2}{2}$$

$$\lim_{x \to -3^{+}} f(x) = \frac{2}{2}$$

$$\lim_{x \to -1^{+}} f(x) = \frac{2}{2}$$

$$\lim_{x \to 2^{+}} f(x) = \frac{1}{2}$$

$$\lim_{x \to 2^{+}} f(x) = \frac{1}{2}$$

$$\lim_{x \to 4^{+}} f(x) = \frac{1}{2}$$

$$\lim_{x \to 4^{+}} f(x) = \frac{1}{2}$$

$$f(2) = \frac{3}{2}$$

$$f(4) = \frac{1}{2}$$

- (b) Circle YES or No for each of the following
 - i. Is f continuous at -3? YES (No
 - ii. Is f continuous at -1? YES $\left(\begin{array}{c} NO \end{array}\right)$
 - iii. Is f continuous at 2? YES $\begin{pmatrix} No \end{pmatrix}$
 - iv. Is f continuous at 4? YES NO

2. (5 points) Find the average rate of change of $f(x) = x^2 - 10x$ over the interval [1, 3].

$$\frac{f(3)-f(1)}{3-1} = \frac{3^2-10.3}{2} - \frac{12^2-10.1}{2} = \frac{(9-30)-(1-10)}{2} = \frac{-21-(-9)}{2} = \frac{-21+9}{2} = \frac{-12}{2} = -6.$$

3. (10 points)

(a) State the definition of derivative.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

(b) Find the derivative f'(x) of the function $f(x) = 5x^2 - 4x - 7$ using the definition of derivative.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \left[\frac{5(x+h)^2 - 4(x+h) - 7}{h} \right] - \left[\frac{5x^2 - 4x - 7}{h} \right]$$

$$= \lim_{h \to 0} \frac{5(x^2 + 2hx + h^2) - 4x - 4h - 7x - 6x^2 + 4x + 7x}{h}$$

$$= \lim_{h \to 0} \frac{5x^2 + 10hx + 5h^2 - 4h - 5x^2}{h}$$

$$= \lim_{h \to 0} \frac{10hx + 5h^2 - 4h}{h}$$

$$= \lim_{h \to 0} \frac{h(10x + 5h - 4)}{h}$$

$$= \lim_{h \to 0} (10x + 5h - 4)$$

$$= \lim_{h \to 0} (10x + 5h - 4)$$

$$= 10x + 5x - 4$$

Circle one: 8:50 5:30

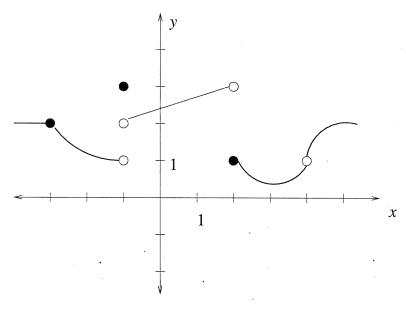
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Quiz 1: Version D

Show your reasoning. Use standard notation correctly.

1. (10 points) The function f is depicted below.



$$\lim_{x \to -3^{-}} f(x) = \frac{2}{2}$$

$$\lim_{x \to -3^{+}} f(x) = \frac{2}{2}$$

$$\lim_{x \to -3^{+}} f(x) = \frac{2}{2}$$

$$\lim_{x \to -1^{+}} f(x) = \frac{2}{2}$$

$$\lim_{x \to 2^{+}} f(x) = \frac{1}{2}$$

$$\lim_{x \to 2^{+}} f(x) = \frac{1}{2}$$

$$\lim_{x \to 4^{+}} f(x) = \frac{1}{2}$$

$$\lim_{x \to 4^{-}} f(x) = \frac{1}{2}$$

- (b) Circle YES or No for each of the following.
 - i. Is f continuous at -3? YES NO
 - ii. Is f continuous at -1? YES NO
 - iii. Is f continuous at 2? YES NO
 - iv. Is f continuous at 4? YES $\left(\begin{array}{c} \text{No} \end{array} \right)$

2. (5 points) Find the average rate of change of $f(x) = x^2 - 4x$ over the interval [1, 3].

$$\frac{f(3)-f(1)}{3-1} = \frac{[(3)^2-4(3)]-[1^2-4\cdot1]}{2}$$

$$= \frac{[9-12]-[1-4]}{2}$$

$$= \frac{-3-(-3)}{2} = \frac{0}{2} = 0.$$

3. (10 points)

(a) State the definition of derivative.

(3)
$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

(b) Find the derivative f'(x) of the function $f(x) = 9x^2 - 3x - 2$ using the definition of derivative.

$$\begin{aligned}
\widehat{P} & f(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h \to 0} \frac{[9(x+h)^2 - 3(x+h) - 2] - [9x^2 - 3x - 2]}{h} \\
&= \lim_{h \to 0} \frac{9(x^2 + 2hx + h^2) - 3x - 2h - 2 - 9x^2 + 3x + 2}{h} \\
&= \lim_{h \to 0} \frac{9x^2 + 18hx + 9h^2 - 3h - 9x^2}{h} \\
&= \lim_{h \to 0} \frac{18hx + 9h^2 - 3h}{h} \\
&= \lim_{h \to 0} \frac{h(18x + 9h - 3)}{h} \\
&= \lim_{h \to 0} \frac{(18x + 9h - 3)}{h} \\
&= 18x - 3.
\end{aligned}$$