Quiz 2: Version A

Show your reasoning. Use standard notation correctly. Simplify your answers.
You may NOT share calculators or use a cell phone or any Internet device as a calculator.

(1) (5 pts) An investment of P dollars is made with an annual rate r compounded annually. After 1 year, the amount in the account is

$$P + rP = P(1 + r).$$

DERIVE the formula for the amount in the account after 2 years. (That is, show where this formula comes from.)

amount from + interest
1st year   earned 2nd year

$$[P(1+r)] + r[P(1+r)]$$
$$= [P(1+r)]\{1+r\} \quad \text{factor out GCF}$$
$$= P(1+r)(1+r)$$
$$= P(1+r)^2$$

(2) (5 pts) An investment grows at 3.25% compounded continuously. How many years will it take to triple? *Set up and solve an equation. Show all steps clearly. Give both an exact answer and an approximation rounded to the nearest tenth of a year.*

$$FV = PVe^{0.0325t}$$

what is t when $FV = 3PV$?

$$PVe^{0.0325t} = 3PV$$
$$e^{0.0325t} = 3$$
$$\ln e^{0.0325t} = \ln 3$$
$$0.0325\,t = \ln 3$$
$$t = \frac{\ln 3}{0.0325} \quad (\text{exact})$$

$$t \approx 33.8 \text{ years}$$

(3) (5 pts) Find $\dfrac{dy}{dx}$ where $y = \ln(1 - 2x)$.

$$\frac{dy}{dx} = \frac{1}{1-2x}(-2) = \frac{-2}{1-2x} \quad \text{Chain Rule}$$

(4) (5 pts) Find $\dfrac{d}{dt}(40e^{-25t})$.

$$= 40e^{-25t}(-25) \quad \text{Chain Rule}$$
$$= -1000e^{-25t}$$

(5) (5 pts) Find $f'(x)$ where $f(x) = \dfrac{e^x + 1}{e^x - 1}$.

$$f'(x) = \frac{e^x(e^x - 1) - (e^x + 1)(e^x)}{(e^x - 1)^2} \quad \text{Quotient Rule}$$
$$= \frac{e^x[(e^x - 1) - (e^x + 1)]}{(e^x - 1)^2}$$
$$= \frac{e^x[e^x - 1 - e^x - 1]}{(e^x - 1)^2}$$
$$= \frac{e^x(-2)}{(e^x - 1)^2}$$
$$= \frac{-2e^x}{(e^x - 1)^2}.$$

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Circle one: 8:50   5:30

Intuitive Calculus

Spring 2011
Ms. Kracht

Name: _____KEY_____ Quiz Score: _____/25

Quiz 2: Version B

Show your reasoning. Use standard notation correctly. Simplify your answers.
You may NOT share calculators or use a cell phone or any Internet device as a calculator.

(1) (5 pts) An investment of P dollars is made with an annual rate r compounded annually. After 1 year, the amount in the account is

$$P + rP = P(1+r).$$

DERIVE the formula for the amount in the account after 2 years. (That is, show where this formula comes from.)

$$\left(\begin{array}{c}\text{amount from} \\ \text{1st year}\end{array}\right) + \left(\begin{array}{c}\text{interest} \\ \text{earned 2nd year}\end{array}\right)$$

$$\left[P(1+r) \right] + r\left[P(1+r) \right]$$

$$= \left[P(1+r) \right]\{1 + r\} \quad \begin{array}{c}\text{factor} \\ \text{GCF}\end{array}$$

$$= P(1+r)(1+r)$$

$$= P(1+r)^2$$

(2) (5 pts) An investment grows at 4.75% compounded continuously. How many years will it take to triple? *Set up and solve an equation. Show all steps clearly. Give both an exact answer and an approximation rounded to the nearest tenth of a year.*

$$FV = PVe^{0.0475t}$$

What is t when $FV = 3PV$?

$$PVe^{0.0475t} = 3PV$$

$$e^{0.0475t} = 3$$

$$\ln e^{0.0475t} = \ln 3$$

$$0.0475t = \ln 3$$

$$t = \frac{\ln 3}{0.0475} \quad \text{(exact)}$$

$$t \approx 23.1 \text{ years}$$

(3) (5 pts) Find $\frac{dy}{dx}$ where $y = \ln(1 - 5x)$.

$$\frac{dy}{dx} = \frac{1}{(1-5x)}(-5) = \frac{-5}{1-5x} \quad \begin{array}{c}\text{Chain} \\ \text{Rule}\end{array}$$

(4) (5 pts) Find $\frac{d}{dt}\left(30e^{-15t}\right) = 30e^{-15t}(-15) \quad \begin{array}{c}\text{Chain} \\ \text{Rule}\end{array}$

$$= -450e^{-15t}.$$

(5) (5 pts) Find $f'(x)$ where $f(x) = \frac{e^x + 2}{e^x - 2}$.

$$f'(x) = \frac{e^x(e^x - 2) - (e^x + 2)(e^x)}{(e^x - 2)^2} \quad \begin{array}{c}\text{Quotient} \\ \text{Rule}\end{array}$$

$$= \frac{e^x\left[(e^x - 2) - (e^x + 2)\right]}{(e^x - 2)^2}$$

$$= \frac{e^x\left[e^x - 2 - e^x - 2\right]}{(e^x - 2)^2}$$

$$= \frac{e^x(-4)}{(e^x - 2)^2}$$

$$= \frac{-4e^x}{(e^x - 2)^2}.$$

Name: KEY _____ Quiz Score: ___/25

Quiz 2: Version C

Show your reasoning. Use standard notation correctly. Simplify your answers.
You may NOT share calculators or use a cell phone or any Internet device as a calculator.

(1) (5 pts) An investment of P dollars is made with an annual rate r compounded annually. After 1 year, the amount in the account is

$$P + rP = P(1 + r).$$

DERIVE the formula for the amount in the account after 2 years. (That is, show where this formula comes from.)

$$\left[\begin{array}{c}\text{amount from}\\ 1^{st}\ \text{year}\end{array}\right] + \left[\begin{array}{c}\text{interest earned}\\ 2^{nd}\ \text{year}\end{array}\right]$$

$$\left[P(1+r)\right] + r\left[P(1+r)\right]$$

$$= \left[P(1+r)\right]\{1 + r\} \quad \text{factor GCF}$$

$$= P(1+r)(1+r)$$

$$= P(1+r)^2 .$$

(2) (5 pts) An investment grows at 5.85% compounded continuously. How many years will it take to triple? *Set up and solve an equation. Show all steps clearly. Give both an exact answer and an approximation rounded to the nearest tenth of a year.*

$$FV = PV\, e^{0.0585t}$$

What is t when $FV = 3PV$?

$$PV\, e^{0.0585t} = 3PV$$

$$e^{0.0585t} = 3$$

$$\ln e^{0.0585t} = \ln 3$$

$$0.0585\, t = \ln 3$$

$$t = \frac{\ln 3}{0.0585} \quad \text{(exact)}$$

$$t \approx 18.8 \text{ years}$$

(3) (5 pts) Find $\dfrac{dy}{dx}$ where $y = \ln(1 - 7x)$.

$$\frac{dy}{dx} = \frac{1}{1-7x}\,(-7) = \frac{-7}{1-7x}$$

Chain Rule

(4) (5 pts) Find $\dfrac{d}{dt}\left(50e^{-15t}\right) = 50\, e^{-15t}\,(-15)$

$$= -750\, e^{-15t}$$

(5) (5 pts) Find $f'(x)$ where $f(x) = \dfrac{e^x + 3}{e^x - 3}$.

$$f'(x) = \frac{e^x(e^x - 3) - (e^x + 3)(e^x)}{(e^x - 3)^2} \quad \text{Quotient Rule}$$

$$= \frac{e^x\left[(e^x - 3) - (e^x + 3)\right]}{(e^x - 3)^2}$$

$$= \frac{e^x\left[e^x - 3 - e^x - 3\right]}{(e^x - 3)^2}$$

$$= \frac{e^x\left[-6\right]}{(e^x - 3)^2}$$

$$= \frac{-6e^x}{(e^x - 3)^2} .$$

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Quiz 2: Version D

Show your reasoning. Use standard notation correctly. Simplify your answers.
You may NOT share calculators or use a cell phone or any Internet device as a calculator.

(1) (5 pts) An investment of P dollars is made with an annual rate r compounded annually. After 1 year, the amount in the account is

$$P + rP = P(1 + r).$$

DERIVE the formula for the amount in the account after 2 years. (That is, show where this formula comes from.)

$$\left[\begin{array}{c}\text{amount from}\\ \text{1st year}\end{array}\right] + \left[\begin{array}{c}\text{interest earned}\\ \text{during 2nd yr}\end{array}\right]$$

$$\left[P(1+r)\right] + r\left[P(1+r)\right]$$

$$= \left[P(1+r)\right]\{1+r\} \quad \text{factor GCF}$$

$$= P(1+r)(1+r)$$

$$= P(1+r)^2.$$

(2) (5 pts) An investment grows at 6.15% compounded continuously. How many years will it take to triple? *Set up and solve an equation. Show all steps clearly. Give both an exact answer and an approximation rounded to the nearest tenth of a year.*

$$FV = PV\, e^{0.0615t}$$

What is t when $FV = 3PV$?

$$PV\, e^{0.0615t} = 3PV$$

$$e^{0.0615t} = 3$$

$$\ln e^{0.0615t} = \ln 3$$

$$0.0615t = \ln 3$$

$$t = \frac{\ln 3}{0.0615} \quad (\text{exact})$$

$$t \approx 17.9 \text{ years.}$$

(3) (5 pts) Find $\dfrac{dy}{dx}$ where $y = \ln(1 - 3x)$.

$$\frac{dy}{dx} = \frac{1}{1-3x}(-3) = \frac{-3}{1-3x}.$$

(4) (5 pts) Find $\dfrac{d}{dt}\left(20e^{-45t}\right) = 20e^{-45t}(-45)$

$$= -900\, e^{-45t}$$

(5) (5 pts) Find $f'(x)$ where $f(x) = \dfrac{e^x + 9}{e^x - 9}$.

$$f'(x) = \frac{e^x(e^x - 9) - (e^x + 9)e^x}{(e^x - 9)^2}$$

$$= \frac{e^x\left[(e^x - 9) - (e^x + 9)\right]}{(e^x - 9)^2}$$

$$= \frac{e^x\left[e^x - 9 - e^x - 9\right]}{(e^x - 9)^2}$$

$$= \frac{e^x(-18)}{(e^x - 9)^2}$$

$$= \frac{-18e^x}{(e^x - 9)^2},$$