Bayes’ Theorem: Preliminary Exercises
Write complete solutions to all questions, using the proper mathematical notation.

1. Suppose you are charged with a crime. There are three judges that you might appear before: Judge Judy, Judge Rudy, and Judge Trudy. Judges are assigned to cases at random, but Judge Rudy hears twice as many cases as either of the other two judges. If \(J\) is the event you appear before Judge Judy, \(R\) is the event you appear before Judge Rudy, and \(T\) is the event you appear before Judge Trudy, then \(P(J) = 0.25\), \(P(R) = 0.5\), and \(P(T) = 0.25\).

By studying the past decisions of these three judges on related cases, you estimate that your probability of a conviction if you appear before Judge Judy is 0.75, if you appear before Judge Rudy is 0.40, and if you appear before Judge Trudy is 0.52.

Let \(C\) be the event that you are convicted and let \(A\) be the event that you are acquitted (not convicted).

(a) Fill in the blanks to make a true statement.
   \[\begin{align*}
   &\text{i. } P(\_\_\_) = 0.75 \\
   &\text{ii. } P(\_\_\_) = 0.40 \\
   &\text{iii. } P(\_\_\_) = 0.52
   \end{align*}\]

(b) Draw a tree diagram for the situation described. Label each edge with the corresponding probability.

(c) Describe each event in plain English in terms of judges, convictions, etc. (Do not use mathematical terms such as “intersection.”)
   \[\begin{align*}
   &\text{i. } J \cap C \\
   &\text{ii. } J \cap A \\
   &\text{iii. } R \cap C \\
   &\text{iv. } R \cap A \\
   &\text{v. } T \cap C \\
   &\text{vi. } T \cap A
   \end{align*}\]

(d) Compute each probability. \textit{Hint: Use your tree diagram.}
   \[\begin{align*}
   &\text{i. } P(J \cap C) \\
   &\text{ii. } P(J \cap A) \\
   &\text{iii. } P(R \cap C) \\
   &\text{iv. } P(R \cap A) \\
   &\text{v. } P(T \cap C) \\
   &\text{vi. } P(T \cap A)
   \end{align*}\]

(e) Are the events \(J \cap C\), \(R \cap C\), and \(T \cap C\) mutually exclusive? \textit{Explain why or why not.}

(f) Is \(C = (J \cap C) \cup (R \cap C) \cup (T \cap C)\)? \textit{Explain why or why not.}

(g) Find the probability that you will be convicted. \textit{Explain why you can use the formula you are using.}

2. Your retail business is considering holding a sidewalk sale promotion next Saturday. Past experience indicates that the probability of a successful sale is 0.60, provided it does not rain. This drops to 0.10 if it does rain Saturday. The Weather Channel gives the chance of rain this Saturday to be 20%.

(a) Introduce notation for events relevant to this situation. (“\textit{Let \_ be the event that...}”)

(b) Restate each of the following using probability notation and the events you introduced in the previous part.
   \[\begin{align*}
   &\text{i. The chance of rain on Saturday is 20\%.} \\
   &\text{ii. If it does not rain, the probability of a successful sale is 0.60.} \\
   &\text{iii. If it does rain, the probability of a successful sale is 0.10.}
   \end{align*}\]

(c) Draw a tree diagram for the situation described. Label each edge with the corresponding probability.

(d) Find the probability of each of the following events. \textit{Be sure to use proper notation.}
   \[\begin{align*}
   &\text{i. the event that it will rain and the sale is successful} \\
   &\text{ii. the event that it will not rain and the sale is successful} \\
   &\text{iii. the event that it will rain and the sale is not successful} \\
   &\text{iv. the event that it will not rain and the sale is not successful}
   \end{align*}\]

(e) Are the events “it will rain and the sale is successful” and “it will not rain and the sale is successful” mutually exclusive? \textit{Explain why or why not.}

(f) Find the probability that the sale is successful.
3. A survey of 24,000 people employed in the service industry was taken. It found that 2000 were employed in private households, 1600 were employed as (commercial) security guards, and the rest were employed in other service areas. Suppose 4.6% employed in private households had college degrees, 26.5% employed as security guards had college degrees, and 34% of the rest had college degrees.

(a) Introduce notation for events relevant to this situation. ("Let \( \_ \) be the event that..."")

(b) Find each of the following probabilities, using probability notation and the events you introduced in the previous part.

i. the probability that a person in the service industry is employed in a private household
ii. the probability that a person in the service industry is employed as a security guard
iii. the probability that a person in the service industry is employed in other service areas

(c) Restate each of the following using probability notation and the events you introduced above.

i. 4.6% employed in private households have college degrees
ii. 26.5% employed as security guards have college degrees
iii. 34% employed in other service areas have college degrees

(d) Draw a tree diagram for the situation described. Label each edge with the corresponding probability.

(e) Find the probability of each of the following events. Be sure to use proper notation.

i. the event that a person in the service industry is employed in a private household and has a college degree
ii. the event that a person in the service industry is employed in a private household and does not have a college degree
iii. the event that a person in the service industry is employed as a security guard and has a college degree
iv. the event that a person in the service industry is employed as a security guard and does not have a college degree
v. the event that a person in the service industry is employed in other service areas and has a college degree
vi. the event that a person in the service industry is employed in other service areas and does not have a college degree

(f) Are the events "a person in the service industry is employed in a private household and has a college degree," "a person in the service industry is employed as a security guard and has a college degree," and "a person in the service industry is employed in other service areas and has a college degree" mutually exclusive? Explain why or why not.

(g) Find the probability that a person in the service industry has a college degree.