Conditional Probability II

Write complete solutions to all questions, using the proper mathematical notation.

1. A student must answer a multiple-choice question with five possible answers.
   (a) What is the probability that the student will get the correct answer by guessing?
   (b) What is the probability that the student will get the correct answer by guessing if two choices can be ruled out?

2. A card is drawn from a standard deck of 52 cards. What is the probability that it is an ace given:
   (a) no information
   (b) the card is not a 2
   (c) the card is a heart
   (d) the card is not a face card

3. A card is drawn from a standard deck of 52 cards. What is the probability that the card is less than a 7 given:
   (a) no information
   (b) the card is not a 2
   (c) the card is a heart
   (d) the card is a 3 or 4

4. If a fair coin is flipped three times, what is the probability that it comes up tails at most once given:
   (a) no information
   (b) all three flips produce the same side
   (c) it comes up heads at most once
   (d) it comes up tails at most once
   (e) the third flip is heads
   (f) the third flip is tails
   (g) it comes up heads at least once

5. If a fair coin is flipped three times, what is the probability that it comes up tails at least once given:
   (a) no information
   (b) all three flips produce the same side
   (c) it comes up heads at most once
   (d) it comes up tails at most once
   (e) the third flip is heads
   (f) the third flip is tails
   (g) it comes up heads at least once
6. In a medical study of the common cold, 100 cold sufferers exhibited the following symptoms:

15 people had fevers  
40 people had coughs  
80 people had stuffy noses  
23 people had coughs and stuffy noses  
12 people had fevers and stuffy noses  
4 people had fevers and coughs  
2 people had fevers, coughs, and stuffy noses

(a) Introduce notation for three events relevant to this situation. ("Let __ be the event that…")

(b) Sketch a Venn diagram for the situation, labeling each region appropriately.

(c) Restate each of the following using probability notation and the events you introduced in the previous part.

i. 15 people had fevers  
ii. 40 people had coughs  
iii. 80 people had stuffy noses  
iv. 23 people had coughs and stuffy noses

v. 12 people had fevers and stuffy noses
vi. 4 people had fevers and coughs
vii. 2 people had fevers, coughs, and stuffy noses

(d) If a person randomly selected from the group has a stuffy nose, what is the probability that the person also has a cough? Use proper mathematical notation.

(e) If a person randomly selected from the group has a cough, what is the probability that the person also has a stuffy nose?

(f) If a person randomly selected from the group has a stuffy nose, what is the probability that the person also has a fever?

(g) If a person randomly selected from the group has a fever, what is the probability that the person also has a stuffy nose?

(h) If a person randomly selected from the group has a fever, what is the probability that the person also has a cough?

(i) If a person randomly selected from the group has a cough, what is the probability that the person also has a fever?

(j) If a person randomly selected from the group has a fever and a stuffy nose, what is the probability that the person has a cough?

(k) If a person randomly selected from the group has a cough and a fever, what is the probability that the person has a stuffy nose?

(l) If a person randomly selected from the group has a cough and a stuffy nose, what is the probability that the person does not have a fever?

(m) Are having a cough and having a stuffy nose mutually exclusive events? Explain in mathematical terms.

(n) Are having a cough and having a fever mutually exclusive events? Explain in mathematical terms.

(o) Are having a stuffy nose and having a fever mutually exclusive events? Explain in mathematical terms.

(p) Are having a cough and having a stuffy nose independent events? Explain in mathematical terms.

(q) Are having a cough and having a fever independent events? Explain in mathematical terms.

(r) Are having a stuffy nose and having a fever independent events? Explain in mathematical terms.