Exponential and Logarithmic Functions

1. On a full sheet of graph paper, sketch the graph of \( y = 3^x \). Label intercepts and at least 4 other points with their ordered pairs and the asymptote with the equation of the line. **On the same coordinate system**, sketch the graph of \( y = \log_3 x \). Label intercepts and at least 4 other points with their ordered pairs and the asymptote with the equation of the line.

2. On a full sheet of graph paper, sketch the graph of \( y = 4^x \). Label intercepts and at least 4 other points with their ordered pairs and the asymptote with the equation of the line. **On the same coordinate system**, sketch the graph of \( y = \log_4 x \). Label intercepts and at least 4 other points with their ordered pairs and the asymptote with the equation of the line.

3. On a full sheet of graph paper, sketch the graph of \( y = e^x \). Label intercepts and at least 4 other points with their ordered pairs and the asymptote with the equation of the line. **On the same coordinate system**, sketch the graph of \( y = \ln x \) (= \( \log_e x \)). Label intercepts and at least 4 other points with their ordered pairs and the asymptote with the equation of the line.

4. For each logarithmic equation, write the equivalent exponential equation.

   **Example.** \( \log_5 25 = 2 \) is equivalent to \( 5^2 = 25 \).

   **Example.** \( \ln z = 5 \) is equivalent to \( e^5 = z \).

   (a) \( \log_7 49 = 2 \)  
   (b) \( \log_7 1 = 0 \)  
   (c) \( \log_{10} 0.1 = -1 \)  
   (d) \( \log_{36} 6 = \frac{1}{2} \)  
   (e) \( \ln 2 = x \)  
   (f) \( \ln y = 17 \)

5. For each exponential equation, write the equivalent logarithmic equation.

   **Example.** \( 5^{-1} = \frac{1}{5} \) is equivalent to \( \log_5 \left( \frac{1}{5} \right) = -1 \).

   **Example.** \( e^x = 10 \) is equivalent to \( \ln 10 = x \).

   (a) \( 5^4 = 625 \)  
   (b) \( 10^{-3} = 0.001 \)  
   (c) \( 125^{1/3} = 5 \)  
   (d) \( \left( \frac{1}{4} \right)^{-2} = 16 \)  
   (e) \( e^{0.025} = g \)  
   (f) \( e^M = Q \)

6. Evaluate each expression without using a calculator or computer (other than your brain!).

   **Example.** \( \log_{15} \left( \frac{1}{15} \right) = -1 \)

   **Example.** \( e^{\ln 37} = 37 \)

   (a) \( \log_5 5 \)  
   (b) \( \log_{17} 1 \)  
   (c) \( \log_2 32 \)  
   (d) \( \log_2 \left( \frac{1}{16} \right) \)  
   (e) \( \log_2 \sqrt{2} \)  
   (f) \( 2^{\log_2 5} \)  
   (g) \( \ln e \)  
   (h) \( \ln e^3 \)  
   (i) \( \ln 1 \)  
   (j) \( \ln \left( \frac{1}{e} \right) \)  
   (k) \( \ln \sqrt{e} \)  
   (l) \( e^{\ln 1.023} \)