Stock Options: Payoff Diagrams and Hedging

Please do the assignment on lined loose leaf paper. Use graph paper for the graphs.

Let $A$ (for asset) denote the value (share price) of a stock on a certain date. Let $V(A)$ denote the value of the option on its expiration date. As the notation suggests, the value of the option is a function of the value $A$ of the underlying asset at that time.

1. Consider a European call option on ABC stock with an exercise (strike) price of $150 per share.
   
   (a) Compute each of the following.
   
   i. $V(0)$
   
   ii. $V(25)$
   
   iii. $V(50)$
   
   iv. $V(100)$
   
   v. $V(150)$
   
   vi. $V(200)$
   
   vii. $V(250)$
   
   viii. $V(300)$
   
   (b) Sketch the payoff diagram for this call option. That is, sketch the graph of $V(A)$. Hint: Plot and label the points you found above. Your graph should be at least a half-sheet of graph paper.
   
   (c) Express $V(A)$ as a function of $A$ using the maximum function ($\max(x, y)$ gives as its output the maximum of the numbers $x$ and $y$). Then express it as a piece-wise defined function.

2. Consider a European call option on XYZ stock with an exercise (strike) price of $30 per share.
   
   (a) Compute each of the following.
   
   i. $V(0)$
   
   ii. $V(5)$
   
   iii. $V(10)$
   
   iv. $V(20)$
   
   v. $V(30)$
   
   vi. $V(40)$
   
   vii. $V(50)$
   
   viii. $V(60)$
   
   (b) Sketch the payoff diagram for this call option. That is, sketch the graph of $V(A)$. Hint: Plot and label the points you found above. Your graph should be at least a half-sheet of graph paper.
   
   (c) Express $V(A)$ as a function of $A$ using the maximum function. Then express it as a piece-wise defined function.

3. Consider a European call option on STU stock with an exercise (strike) price of $8 per share.
   
   (a) Compute each of the following.
   
   i. $V(0)$
   
   ii. $V(2)$
   
   iii. $V(4)$
   
   iv. $V(6)$
   
   v. $V(8)$
   
   vi. $V(10)$
   
   vii. $V(12)$
   
   viii. $V(14)$
   
   (b) Sketch the payoff diagram for this call option. That is, sketch the graph of $V(A)$. Hint: Plot and label the points you found above. Your graph should be at least a half-sheet of graph paper.
   
   (c) Express $V(A)$ as a function of $A$ using the maximum function. Then express it as a piece-wise defined function.

4. Consider a European put option on ABC stock with an exercise (strike) price of $150 per share.
   
   (a) Compute each of the following.
   
   i. $V(0)$
   
   ii. $V(25)$
   
   iii. $V(50)$
   
   iv. $V(100)$
   
   v. $V(150)$
   
   vi. $V(200)$
   
   vii. $V(250)$
   
   viii. $V(300)$
(b) Sketch the payoff diagram for this put option. That is, sketch the graph of \( V(A) \). Hint: Plot and label the points you found above. Your graph should be at least a half-sheet of graph paper.

(c) Express \( V(A) \) as a function of \( A \) using the maximum function. Then express it as a piece-wise defined function.

5. Consider a European put option on XYZ stock with an exercise (strike) price of $30 per share.

(a) Compute each of the following.

\[
\begin{align*}
&i. \ V(0) \quad \text{iii.} \ V(10) \quad \text{v.} \ V(30) \quad \text{vii.} \ V(50) \\
&ii. \ V(5) \quad \text{iv.} \ V(20) \quad \text{vi.} \ V(40) \quad \text{viii.} \ V(60)
\end{align*}
\]

(b) Sketch the payoff diagram for this put option. That is, sketch the graph of \( V(A) \). Hint: Plot and label the points you found above. Your graph should be at least a half-sheet of graph paper.

(c) Express \( V(A) \) as a function of \( A \) using the maximum function. Then express it as a piece-wise defined function.

6. Consider a European put option on STU stock with an exercise (strike) price of $8 per share.

(a) Compute each of the following.

\[
\begin{align*}
&i. \ V(0) \quad \text{iii.} \ V(4) \quad \text{v.} \ V(8) \quad \text{vii.} \ V(12) \\
&ii. \ V(2) \quad \text{iv.} \ V(6) \quad \text{vi.} \ V(10) \quad \text{viii.} \ V(14)
\end{align*}
\]

(b) Sketch the payoff diagram for this put option. That is, sketch the graph of \( V(A) \). Hint: Plot and label the points you found above. Your graph should be at least a half-sheet of graph paper.

(c) Express \( V(A) \) as a function of \( A \) using the maximum function. Then express it as a piece-wise defined function.

7. **Hedging** is the practice of combining financial instruments whose values move in opposite directions with a change in the value of an underlying asset in order to reduce risk. This is done with various combinations of shares of stock, call options, and put options on the same stock. Consider a portfolio consisting on one call option and one put option on LMN stock, each with an exercise price of $50 per share and both with the same expiration date. Let \( V(A) \) denote the value of the portfolio on the expiration date where \( A \) is the value of the underlying asset (LMN stock).

(a) Compute each of the following.

\[
\begin{align*}
&i. \ V(0) \quad \text{iv.} \ V(30) \quad \text{vii.} \ V(60) \quad \text{x.} \ V(90) \\
&ii. \ V(10) \quad \text{v.} \ V(40) \quad \text{viii.} \ V(70) \\
&iii. \ V(20) \quad \text{vi.} \ V(50) \quad \text{ix.} \ V(80) \quad \text{xi.} \ V(100)
\end{align*}
\]

(b) Sketch the payoff diagram for this portfolio. That is, sketch the graph of \( V(A) \). Hint: Plot and label the points you found above. Your graph should be at least a half-sheet of graph paper.

(c) Express \( V(A) \) as a function of \( A \) using the maximum function. Then express it as a piece-wise defined function.