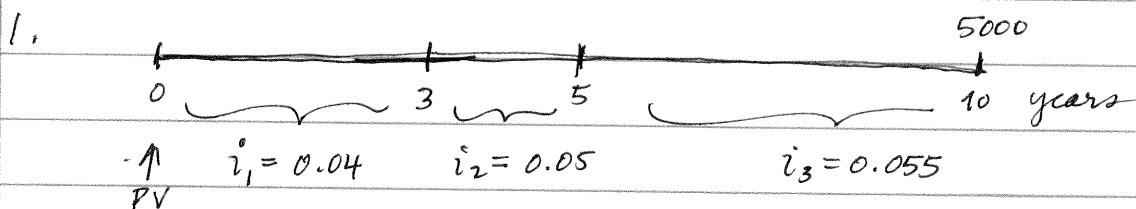


Mathematical Theory of Interest

Equations of Value Worksheet Solutions



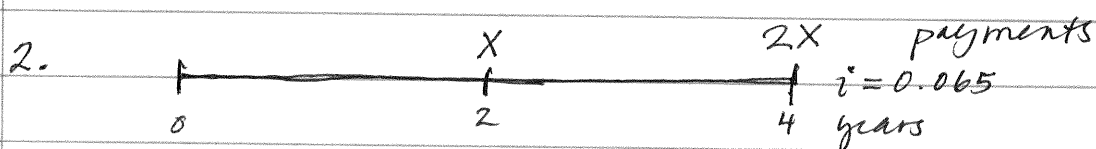
We must discount the \$5000 at time 10 back to time zero. Use the discount factors:

$$v_1 = \frac{1}{1.04}, \quad v_2 = \frac{1}{1.05}, \quad v_3 = \frac{1}{1.055}$$

$$\text{Then } PV = 5000 v_3^5 v_2^2 v_1^3$$

$$= 5000 (1.055)^{-5} (1.05)^{-2} (1.04)^{-3}$$

$$\approx \underline{\underline{\$ 3084.81}}$$



debt \$6000

$$v = \frac{1}{1.065}$$

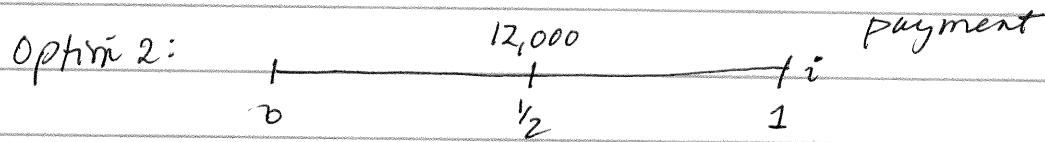
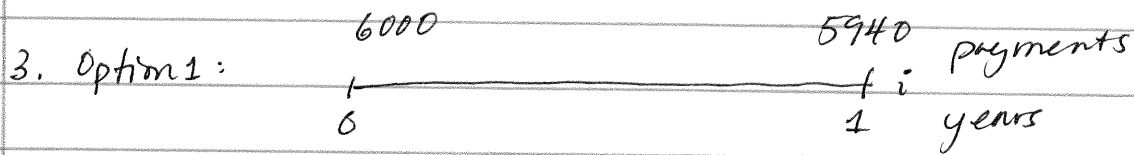
Equation of value at time 0:

$$Xv^2 + 2Xv^4 = 6000$$

$$X(v^2 + 2v^4) = 6000$$

$$X = \frac{6000}{v^2 + 2v^4}$$

$$= \underline{\underline{\$ 2,462.75}}$$



$$PV_1 = 6000 + 5940v \quad , \quad \text{where } v = \frac{1}{1+i}$$

$$PV_2 = 12000v^{1/2}$$

Set these equal and solve for i :

$$6000 + 5940v = 12000v^{1/2}$$

$$6000 + 5940v - 12,000v^{1/2} = 0$$

$$100 + 99v - 200v^{1/2} = 0$$

$$100 + \frac{99}{1+i} - \frac{200}{(1+i)^{1/2}} = 0$$

$$100(1+i) + 99 - 200(1+i)^{1/2} = 0$$

$$\text{let } x = (1+i)^{1/2}$$

$$100x^2 - 200x + 99 = 0$$

By the quadratic formula:

$$x = \frac{-(-200) \pm \sqrt{(-200)^2 - 4(100)(99)}}{2(100)}$$

$$= \frac{200 \pm \sqrt{400(400 - 99)}}{200}$$

$$= \frac{200 \pm 20}{200} = \frac{10 \pm 1}{10}$$

$$\text{So } x = \frac{10+1}{10} = 1.1$$

$$\text{or } x = \frac{10-1}{10} = 0.9$$

$$(1+i)^{1/2} = 1.1$$

$$1+i = 1.21$$

$$i = 0.21$$

$$= \underline{\underline{21\%}}$$

$$(1+i)^{1/2} = 0.9$$

$$1+i = 0.81$$

$$i = -0.19$$

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we don't allow
negative interest
in this course