$\frac{24}{25}$ good

MATH 30055 **SPRING 2015** **HW 1** **NAME:_____**

DUE TUESDAY, 3 FEBRUARY 2015

2. (a) Prove that $A(n) - A(0) = I_1 + I_2 + \cdots + I_n$.

SOLUTION.

Since, by definition, $I_n = A(n) - A(n-1)$,

$$\begin{aligned}
&I_n + I_{n-1} + \cdots + I_2 + I_1 \\
&= (A(n) - \cancel{A(n-1)}) + (\cancel{A(n-1)} - \cancel{A(n-2)}) + (\cancel{A(n-2)} - \cancel{A(n-3)}) \\
&+ \cdots + (\cancel{A(2)} - \cancel{A(1)}) + (\cancel{A(1)} - A(0)) \\
&= A(n) - A(0).
\end{aligned}$$

+5 good

(b) Verbally interpret the result obtained in (a).

SOLUTION.

One could calculate the total amount of interest occurred by either subtracting the amount at the beginning of the year from the end of the year or adding up the interests occurred at every period.

7. Show that $A(n) = (1 + i_n)A(n-1)$, where n is a positive integer.

SOLUTION.

Since $i_n = \frac{A(n)-A(n-1)}{A(n-1)}$,

$$\begin{aligned}
(1 + i_n)A(n-1) &= \left(1 + \frac{A(n) - A(n-1)}{A(n-1)}\right) A(n-1) \\
&= \left(\frac{\cancel{A(n-1)} + A(n) - \cancel{A(n-1)}}{\cancel{A(n-1)}}\right) \cancel{A(n-1)} \\
&= A(n).
\end{aligned}$$

+5

So, $A(n) = (1 + i_n)A(n-1)$.

11. Simple interest of $i = 4\%$ is being credited to a fund. In which period is this equivalent to an effective rate of 2 1/2%?

SOLUTION.

The effective rate of interest in simple interest is

why? what is this?

$$\begin{aligned}
0.025 &= \frac{0.04}{1 + 0.04(t-1)} \\
0.025(1 + 0.04t - 0.04) &= 0.04 \\
0.96 + 0.04t &= 1.6 \\
0.04t &= 0.64 \\
t &= \boxed{16 \text{ periods.}}
\end{aligned}$$

+4

14. Show that the ratio of the accumulated value of 1 invested at the rate i for n periods, to the accumulated value of 1 invested at rate j for n periods, $i > j$, is equal to the accumulated value of 1 invested for n periods at rate r. Find an expression for r as a function of i and j.

SOLUTION.

$$\begin{aligned}
(1+r)^n &= \frac{(1+i)^n}{(1+j)^n} \\
1+r &= \sqrt[n]{\frac{(1+i)^n}{(1+j)^n}} \\
1+r &= \frac{1+i}{1+j} \\
r &= \frac{1+i-(1+j)}{1+j} \\
r &= \boxed{\frac{i-j}{1+j}.}
\end{aligned}$$

✓ +5

17. Two sets of grandparents for a newborn baby wish to invest enough money immediately to pay \$10,000 per year for four years toward college costs starting at age 18. Grandparents A agree to fund the first two payments, while Grandparents B agree to fund the last two payments. If the effective rate of interest is 6% per annum, find the difference between the contributions of Grandparents A and B.

SOLUTION.

Grandparents A will need to invest \$10,000 after 18 years have past and \$10,000 after 19 years have past. The present value of this investment would be

$$10,000\left(\frac{1}{(1+0.06)^{18}}\right)+10,000\left(\frac{1}{(1+0.06)^{19}}\right)=\$6,808.57.$$

✓

Grandparents A will need to invest \$10,000 after 20 years have past and \$10,000 after 21 years have past. The present value of this investment would be

$$10,000\left(\frac{1}{(1+0.06)^{20}}\right)+10,000\left(\frac{1}{(1+0.06)^{21}}\right)=\$6,059.60.$$

✓

The difference between these contributions is

$$6,808.57-6,059.60=\boxed{\$748.97.}$$

✓ +5