MATH 30055 | The Mathematical Theory of Interest | Dr. Kracht

Name (print neatly): _____ Score: _____/70

Hand-in Homework 3

Chapter 3: Basic annuities
Chapter 4: General annuities
due: Monday, April 6, 2015

Submit answers on this sheet. Staple pages together.

To receive full credit, show all work necessary to justify answers and all steps of proofs and derivations clearly, in logical sequence, using notation developed in class. Reference formulas and theorems from the book explicitly where needed. Partial credit will be given only for significant progress toward a solution.

---

1. To accumulate 8000 at the end of $3n$ years, deposits of 98 are made at the end of each of the first n years and 196 at the end of each of the the next $2n$ years. The annual effective rate of interest is i. You are given that $(1+i)^n = 2.0$. Find i.

2. A perpetuity-immediate pays X per year. Brian receives the first n payments; Colleen receives the next n payments; and Jeff receives the remaining payments. Brian's share of the present value of the original perpetuity is 40% and Jeff's share is K. Calculate K.

3. (a) Describe in words what the difference $a_{\overline{n+1}|i} - a_{\overline{n}|i}$ is measuring.

   (b) Given that $a_{\overline{n+1}|i} - a_{\overline{n}|i} = 0.177208656$ and $\ddot{a}_{\overline{n+1}|i} - \ddot{a}_{\overline{n}|i} = 0.185248436$, find the integer n.
   Hint: Use your answer to part (a).

4. Osborne received a ten-year annuity. It paid $\$500$ at the end of each quarter for the first four years, and then $\$150$ per month for the remaining six years. Express the value at the time of the last payment in terms of the annuity symbols from §4.4 if the annual effective rate of interest is i for the first four years and j for the following six years. Evaluate if $i = 4.5\%$ and $j = 5.25\%$.

5. Chad's perpetuity has annual payments. The first payment is for $\$240$ and then payments increase by $\$50$ each year until they become level at $\$790$. Find the value of this perpetuity at the time of the first payment using an annual effective rate of interest of 3.75%.

6. On January 1, 2000, Linda inherited a perpetuity with annual payments beginning in one year. The first payment was $\$5000$ and after that, payments increase by 4% each year. Find the value of this perpetuity immediately after the January 1, 2013 payment if the annual effective interest rate is 3% from January 1, 2000 through January 1, 2015 and 5% thereafter.

7. Ted has just inherited an annuity paying Q at the end of the first year, $2Q$ at the end of the second year, and in general, kQ at the end of the kth year, for n years. Show that the present value of this increasing annuity is given by

$$PV = Q(Ia)_{\overline{n}|i}$$

by using the same method that we used in class to derive the formula for $(Ia)_{\overline{n}|i}$. *(This is not the most efficient method, but I want you to practice this approach.)*