MATH 30055 | The Mathematical Theory of Interest | Dr. Kracht

Name (print neatly): Key Score: /20

Quiz 3: Thursday, February 5, 2014

To receive full credit, show all work necessary to justify answers and all steps of solutions and derivations clearly, in logical sequence, using notation developed in class. Partial credit will be given only for significant progress toward a solution.

1. (6 pts) Prove that $a(n) = (1 + i_n)\, a(n-1)$, where n is a positive integer.

Proof. Let n be a positive integer. Then

$$\begin{aligned} \text{LHS} &= (1+i_n)\,a(n-1) \\ &= \left[1 + \frac{a(n)-a(n-1)}{a(n-1)}\right] a(n-1), \text{ by def}^{n} \text{ of } i_n \\ &= a(n-1) + a(n) - a(n-1), \text{ by distributive law} \\ &= a(n) \\ &= \text{RHS}. \end{aligned}$$

□

Given: Sps $a(t) = 1+it$.

2. (6 pts) At what rate of simple interest will \$2000 accumulate to \$2735 in 5.25 years?

What is i so that $2000\,a(5.25) = 2735$?

$$\begin{aligned} 2000(1 + i(5.25)) &= 2735 \\ 1 + 5.25\,i &= \frac{2735}{2000} \\ i &= \left(\frac{2735}{2000} - 1\right)/5.25 \\ i &= 0.07. \end{aligned}$$

3. (8 pts) At a certain rate, i, of compound interest, 1 will increase to 7 in a years, 2 will increase to 10 in b years, and 3 will increase to 6 in c years. If 7 will increase to 10 in n years, find an expression for n in terms of a, b and c.

Sps. $a(t) = (1+i)^t$.

Given $(1+i)^a = 7$

$2(1+i)^b = 10 \Rightarrow (1+i)^b = 5$

$3(1+i)^c = 6 \Rightarrow (1+i)^c = 2$

Now $7(1+i)^n = 10 \Rightarrow (1+i)^n = \frac{10}{7}$

$$= \frac{5\cdot 2}{7} = \frac{(1+i)^b (1+i)^c}{(1+i)^a}$$

So $(1+i)^n = (1+i)^{b+c-a}$

Since $y = (1+i)^t$ is 1-1, $n = b + c - a$.