

Name (print neatly): KEY Score: /20

Quiz 6: Thursday, February 26, 2015

To receive full credit, show all work necessary to justify answers and all steps of proofs and derivations clearly, in logical sequence, using notation developed in class. Partial credit will be given only for significant progress toward a solution.

1. (8 pts) Give the standard formulas (the fractions in terms of i, d , and/or v) for each of the following annuity symbols.

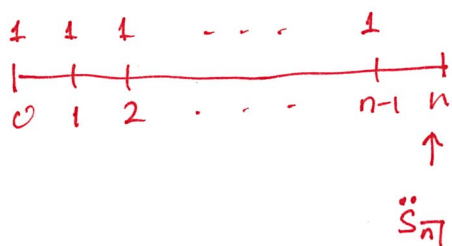
(a) $a_{\overline{n}|i} = \frac{1-v^n}{i}$

(c) $\ddot{a}_{\overline{n}|i} = \frac{1-v^n}{d}$

(b) $s_{\overline{n}|i} = \frac{(1+i)^n - 1}{i}$

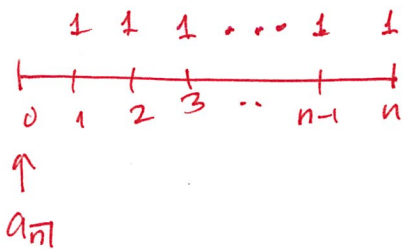
(d) $\ddot{s}_{\overline{n}|i} = \frac{(1+i)^n - 1}{d}$

2. (4 pts) Draw a timeline for $\ddot{s}_{\overline{n}|i}$ and then state what the symbol represents in plain English.



$\ddot{s}_{\overline{n}|i}$ represents the accumulated value, at time n , of an n -year annuity with unit payments at the beginning of each year.

3. (8 pts) Derive the formula for $a_{\overline{n}|i}$ using a timeline and the geometric sum formula.



$$\begin{aligned} a_{\overline{n}|i} &= v + v^2 + v^3 + \dots + v^{n-1} + v^n \\ &= v(1 + v + v^2 + \dots + v^{n-1}) \\ &= v \left(\frac{1-v^n}{1-v} \right) \\ &= \frac{1-v^n}{\frac{1}{v}(1-v)} \\ &= \frac{1-v^n}{\frac{1}{v} - 1} \\ &= \frac{1-v^n}{1+i-1} = \frac{1-v^n}{i} \end{aligned}$$

Geometric Sum:

$$1+r+r^2+\dots+r^{n-1} = \frac{1-r^n}{1-r} \quad (r \neq 1)$$