MATH 11022–006
TR 11:00

Trigonometry

Dr. Kracht
Spring 2014

Name: KEY

Exam Score: ______ /100

105 pts available

Exam 3 Part I: NO CALCULATORS.

You must turn this part in before picking up your calculator.

1. (24 pts) Give the exact value of each of the following or state "undefined."

   (a) $\sin^{-1}\left(-\frac{1}{\sqrt{2}}\right) =$ $-\pi/4$

   (b) $\cos^{-1}(0) =$ $\pi/2$

   (c) $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) =$ $-\frac{\pi}{6}$

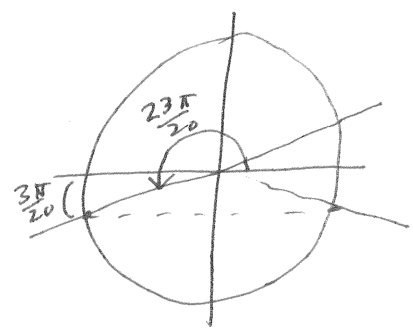
   (d) $\cos^{-1}(0) =$ $\pi/2$

   (e) $\sin\left(\sin^{-1}\left(\frac{20}{19}\right)\right) =$ undef as $\frac{20}{19} > 1$

   (f) $\cos\left(\cos^{-1}\left(\frac{19}{20}\right)\right) =$ $\frac{19}{20}$

   (g) $\tan\left(\tan^{-1}\left(-\frac{45}{2}\right)\right) =$ $-\frac{45}{2}$

   (h) $\sin^{-1}\left(\sin\left(\frac{23\pi}{20}\right)\right) =$ $-\frac{3\pi}{20}$

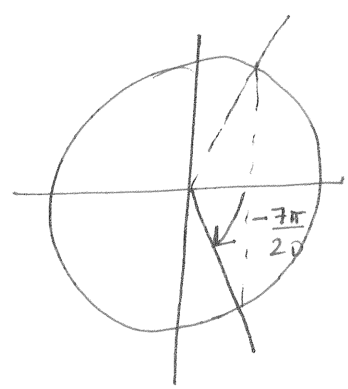


   (i) $\cos^{-1}\left(\cos\left(\frac{3\pi}{20}\right)\right) =$ $\frac{3\pi}{20}$

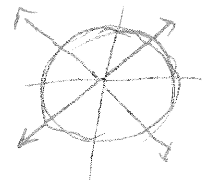
   (j) $\sin^{-1}\left(\sin\left(-\frac{7\pi}{20}\right)\right) =$ $-\frac{7\pi}{20}$

   (k) $\cos^{-1}\left(\cos\left(-\frac{7\pi}{20}\right)\right) =$ $\frac{7\pi}{20}$

   (l) $\tan^{-1}\left(\tan\left(-\frac{7\pi}{20}\right)\right) =$ $-\frac{7\pi}{20}$



2. (10 pts) Find all real solutions, in radians. $\tan^2(3x) - 1 = 0$



$\tan^2(3x) = 1$

$\tan(3x) = \pm 1$

$\tan(3x) = -1$ or $\tan(3x) = 1$

$3x = \frac{\pi}{4} + n\pi$ $\qquad$ $3x = \frac{3\pi}{4} + n\pi$

$\frac{1}{3}(3x) = \frac{1}{3}\left(\frac{\pi}{4} + n\pi\right)$ $\qquad$ $\frac{1}{3}(3x) = \frac{1}{3}\left(\frac{3\pi}{4} + n\pi\right)$

$\underline{x = \frac{\pi}{12} + n\frac{\pi}{3}}$ $\qquad$ $\underline{x = \frac{\pi}{4} + \frac{\pi}{3}n}$

(where $n \in \mathbb{Z}$)

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3. (10 pts) Find all real solutions, in radians. $\sin(2x) - \sin x = 0$

$2\sin x \cos x - \sin x = 0$

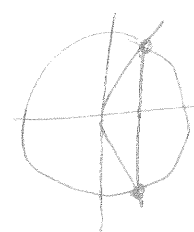
$\sin x\,(2\cos x - 1) = 0$

$\sin x = 0$ or $2\cos x - 1 = 0$

$2\cos x = 1$

$\underline{x = n\pi}$

$\cos x = \frac{1}{2}$



$\underline{x = \frac{\pi}{3} + 2n\pi}$

$\underline{x = \frac{-\pi}{3} + 2n\pi}$

(where $n \in \mathbb{Z}$)

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Exam 3 Part II: Calculators permitted.

No sharing of calculators.

4. (20 points) Use the Law of Sines or the Law of Cosines to find the requested value. If two solutions exist, find both. If no solution exists, explain why not. Assume triangles are labeled in the standard way (angle A opposite side a, etc.). Sketch and label a triangle for each solution, roughly to scale. *Round your answers to the nearest degree. Justify your answers.*

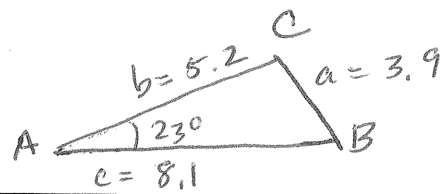
(a) Find a if $b = 5.2$, $c = 8.1$, and $A = 23°$.

$a^2 = b^2 + c^2 - 2bc\cos A$ Law of Cosines

$a^2 = 5.2^2 + 8.1^2 - 2(5.2)(8.1)\cos 23°$

$a > 0$ so $a = +\sqrt{5.2^2 + 8.1^2 - 2(5.2)(8.1)\cos 23°} \approx \sqrt{15.1067\ldots}$

$a \approx \underline{\underline{3.9}}$



(b) Find B if $C = 41°$, $b = 10$, $c = 13$.

$\dfrac{\sin B}{b} = \dfrac{\sin C}{c}$

$\sin B = \dfrac{10 \sin 41°}{13}$

$B = \sin^{-1}\left(\dfrac{10\sin 41°}{13}\right)$

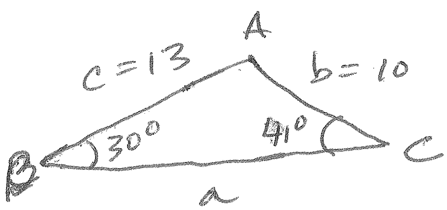
$\approx \underline{\underline{30°}}$

or (possibly) $B = 180° - \sin^{-1}\left(\dfrac{10\sin 41°}{13}\right)$

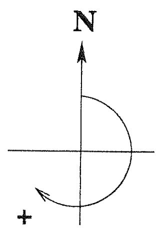
$B \approx 150°$

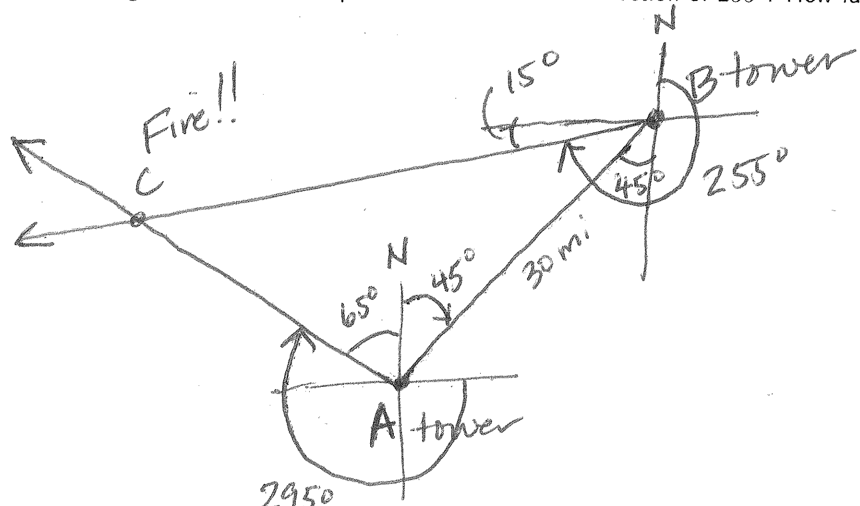
but then $C + B = 41° + 150°$
$= 191° > 180°$

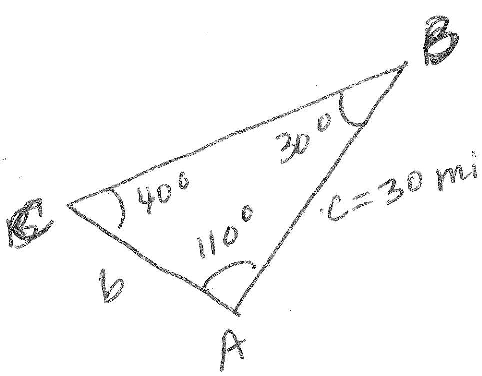
$\cancel{\#}$



5. (15 points) Fire tower B is located 30 miles at a direction of 45° from fire tower A. A ranger in fire tower A spots a fire at a direction of 295°. A ranger in fire tower B spots the same fire at a direction of 255°. How far from tower A is the fire?







$B = 90° - 15° - 45° = \underline{30°}$

$A = 65° + 45° = \underline{110°}$

so $C = 180° - 30° - 110° = \underline{40°}$

Law of Sines $\dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

$b = \dfrac{30 \sin 30°}{\sin 40°} \approx \underline{23 \text{ mi}}$

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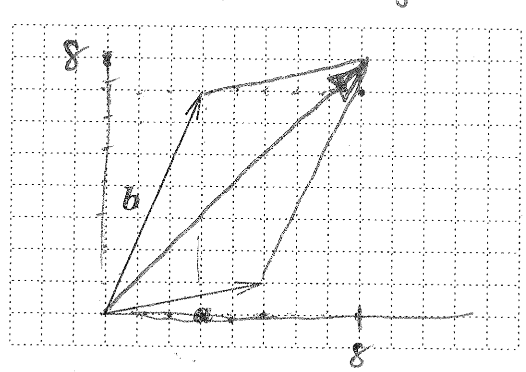
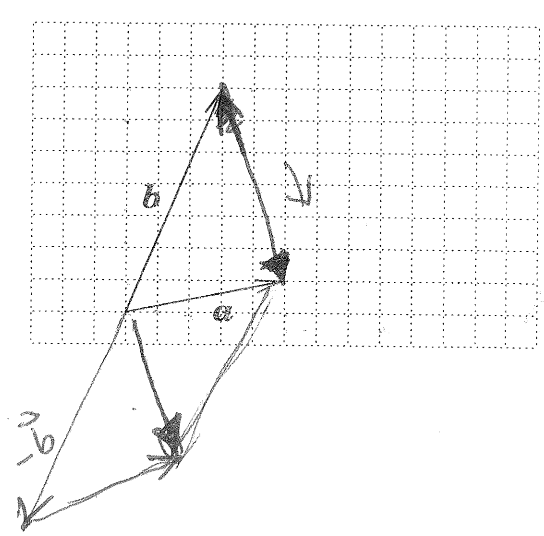
6. (6 points) Sketch each of the requested vectors.

(a) Sketch $\vec{a} + \vec{b}$ parallelogram rule



(b) Sketch $\vec{a} - \vec{b}$



7. (10 points) If $\vec{u} = < 5, -2 >$ and $\vec{v} = < -4, 7 >$, evaluate each of the following.

(a) $\vec{u} + \vec{v} =$ $\underline{\langle 1, 5 \rangle}$

(b) $|\vec{u}| =$ $\underline{\sqrt{29}}$

$$|\vec{u}| = \sqrt{5^2 + (-2)^2} = \sqrt{25+4} = \sqrt{29}$$

(c) $5\vec{u} - 3\vec{v} =$ $\underline{\langle 37, -31 \rangle}$

$$5\langle 5, -2\rangle - 3\langle -4, 7\rangle = \langle 25, -10\rangle + \langle 12, -21\rangle = \langle 37, -31\rangle$$

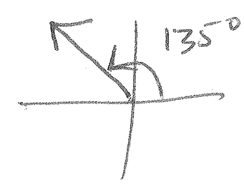
(d) $\frac{\vec{u}}{|\vec{u}|} =$ $\underline{\left\langle \frac{5}{\sqrt{29}}, \frac{-2}{\sqrt{29}} \right\rangle}$

$$\frac{\langle 5, -2\rangle}{\sqrt{29}}$$

(e) $\vec{u} \cdot \vec{v} =$ $\underline{-34}$

$$\vec{u} \cdot \vec{v} = 5\cdot(-4) + (-2)(7) = -20 - 14 = -34$$

8. (10 points) Suppose $|\vec{w}| = 10$ and the direction angle of $\vec{w}$ is $135°$.



(a) Write $\vec{w}$ in component form.

$$\vec{w} = 10\cos 135^\circ\, \vec{i} + 10 \sin 135^\circ\, \vec{j}$$
$$= 10\left(-\frac{1}{\sqrt{2}}\right)\vec{i} + 10\left(\frac{1}{\sqrt{2}}\right)\vec{j}$$
$$= \frac{-10}{\sqrt{2}}\vec{i} + \frac{10}{\sqrt{2}}\vec{j}$$
$$= \left\langle \frac{-10}{\sqrt{2}}, \frac{10}{\sqrt{2}} \right\rangle = \langle -5\sqrt{2}, 5\sqrt{2} \rangle$$

(b) Find a unit vector that has the same direction as $\vec{w}$.

$$\frac{\vec{w}}{|\vec{w}|} = \frac{\vec{w}}{10} = \left\langle -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle = -\frac{1}{\sqrt{2}}\vec{i} + \frac{1}{\sqrt{2}}\vec{j}$$