

Name: KEY Quiz Score: _____ /20

Quiz 5

1. (10 pts)

(a) State the difference formula for cosine:

$$\cos(u - v) = \cos u \cos v + \sin u \sin v$$

(b) Use your answer above and basic identities to **derive** the familiar sum formula for cosine (the formula for $\cos(u + v)$).
State a reason for each step.

$$\begin{aligned} \cos(u + v) &= \cos(u - (-v)) && \text{algebra} \\ &= \cos u \cos(-v) + \sin u \sin(-v) && \text{difference formula} \\ &= \cos u \cos v + \sin u (-\sin v) && \text{odd/even identities} \\ &= \cos u \cos v - \sin u \sin v \quad \blacksquare && \text{algebra.} \end{aligned}$$

2. (10 pts) Verify the following identity. State a reason for each step.

$$\cos^4 x - \sin^4 x = \cos 2x$$

$$\text{LHS} = \cos^4 x - \sin^4 x \quad \text{given}$$

$$= (\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x)$$

$$= [\cos(2x)] [1]$$

$$= \cos(2x)$$

$$= \text{RHS.} \quad \blacksquare$$

$$\text{algebra: } A^2 - B^2 = (A - B)(A + B)$$

{ double angle identity
+
Pythagorean Identity
algebra