A Better Way of Learning Requires Major Changes in the Teaching Environment

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You would think it would be enough to figure out a better way of helping students learn. Mathematics education, from Kindergarten through Collegiate levels is in pretty bad shape in this country and maybe throughout the world. Sure, many students learn a lot, but for the most part, these are the talented, motivated students. They would learn under just about any educational system. That's not enough. The nature of our society requires, and will increasingly require in the 21^{st} Century, a mathematically literate populace. We need a high percentage of the population able to do mathematics at least at intermediate levels. We need more and more people who can think logically. And we need a teaching force with strong abilities in this difficult, but critical subject. The collegiate mathematics curriculum is not producing enough graduates with sufficient knowledge, understanding and apprecation of mathematics. Of course a big part of the reason is the nature of the students who come to us. Their background is not strong enough, their attitudes are not appropriate and the college environment is not always as conducive to learning as it should be. But these are not the only reasons. A very large part of the problem is that we are using archaic methods of teaching, methods that arose to teach talented, motivated students, methods that are not good enough to help a large portion of the population learn mathematics. So we have to find a better way. We need to develop pedagogical strategies that will be more effective in helping more students learn. Once we do that, these new methods will be widely adopted and we will be well on our way to solve a problem of major social importance.

Right?

Wrong.

There are a lot of people engaged in curriculum reform in collegiate mathematics. Many of them are successfuly designing, implementing, and even testing new, more effective methods of helping students learn mathematics. But there is a reluctance to adopt these methods that is producing strong resistance and keeping even the most clearly effective innovations from becoming a part of standard teaching strategies. There are a lot of reasons for this. One of the most general is a result of what I will call, not too seriously, "Dubinsky's Law of Innovation"

Anything that is really innovative will look pretty awful to most people.

Unfortunately, this reaction occurs not only with other faculty who may be considering whether to adopt new methods, but also to the students who are experiencing a new curriculum. Many people who are developing new pedagogical approaches find that their students don't like them. They express their dislike by giving lower scores on their evaluations and this can have serious consequences for faculty, especially young, untenured faculty. Apparently, all of this occurs even in the face of mounting evidence that these methods are leading to better results in terms of student learning.

This negative reaction by students is the bad news. The good news is that it mainly occurs during and at the end of the course. Some time after the course, even a few months later, it seems that the attitudes of students change and not only do they feel better about the new pedagogy, but their attitudes about mathematics in general are more positive and in a better place than the attitudes of students who took courses with traditional pedagogy.

We don't yet have enough data to fully justify the statements in the preceding paragraph. What we have is a fair amount of data pointing to these assertions and justifying them as serious possibilities. We don't have any data that points in the opposite direction.

My purpose in this article is to explore these assertions in the context of a general approach to curriculum reform that I have been involved with throughout the past 10-12 years. I will begin with an overview of the pedagogical strategy, including the courses in which it has been applied and an indication of the kinds of data that has been gathered regarding student learning of the specific topics which we have investigated. This overview will only be a sketch of what can be said about these matters, but I will include pointers to where more information can be obtained. Next, I will take one of the several components of this pedagogical strategy, cooperative learning, and discuss in a little more detail how we use this approach and some of issues that it raises. Then I will describe some of the results which suggest the assertions I have been making about the long term affect of these courses on the students' study of mathematics and how their attitudes look some time after the course is completed. Finally, I will indicate some conclusions that could be drawn.

Overview of a pedagogical strategy

This pedagogical strategy is part of an overall framework for research and curriculum development in undergraduate mathematics. I will sketch this framework briefly and refer the reader to Asiala et al (1996) for more details. The framework has three components: theoretical analysis, instructional treatment, and data collection/analysis, organized as indicated in Figure 1.

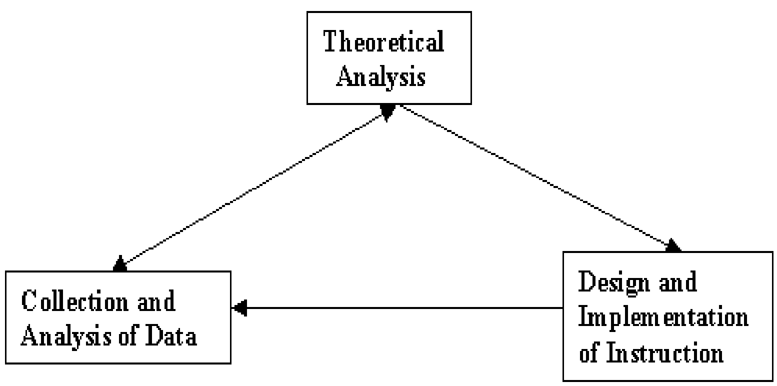

Figure 1

Theoretical analysis

The theoretical analysis is based on a constructivist theory of how learning a mathematical concept might take place. According to this theory, learning mathematics consists of making certain kinds of mental constructs in response to mathematical problem situations. The general theory describes these constructions in terms of what are called actions, processes, objects and schemas. The analysis consists of describing, for the particular mathematical concepts being considered, the specific constructions and their relations.

For example, if the concept in question were cosets, learning would begin with an *action* conception consisting of forming cosets which can be described by listing their elements, such as the cosets of the subgroup of multiples of 4 in the group of integers mod 24 with addition mod 24 as the operation. Calculations could be made of specific cosets and properties such as the number of elements in a coset, the number of cosets of given subgroup, and disjointness could be observed. At this level of understanding, the student would not be able to consider larger and more complicated groups and a binary operation on the set of cosets of a subgroup would be hard to understand. The idea of a quotient group would be essentially inaccessible.

A higher level of understanding would be a *process* conception in which the individual interiorizes the actions. He or she no longer requires explicit listings and calculations to perform operations but can imagine them or run through them mentally. Thus given a group G, a subgroup H, and an element $x \in G$, the individual can think of running through the elements of H and forming the group product of each with x (on the left, say). Not only does this allow the individual to work with more complicated groups, such as the group of permutations of n objects, but he or she can also think about other processes such as figuring out cardinalities and forming the set of all products of two group elements, the first from one specific coset and the second from another.

In moving towards thinking about properties of cosets and performing actions on them, the individual must encapsulate the process understanding and develop an *object* conception of cosets. Then it becomes possible to consider theorems concerning the cardinality of every coset of a particular subgroup, the number of cosets and their disjointness.

At this point, major ideas such as Lagrange's theorem become accessible to the student and he or she can think of constructing a binary operation on the set of cosets of a subgroup, determining its properties and thereby getting to the ideas of normality and quotient groups. In working with these concepts, it is important for the individual to pass easily from the object back to the process from which it came and then return to the object as needed to work with particular situations. Thus, having constructed (mentally) the set of cosets of a subgroup, the individual can think about a binary operation which takes two cosets and produces another coset. Such thoughts require an object conception of cosets. But to actually construct the binary operation (as the set of all products of two elements, one from each coset) the individual must return to the process conception. In many mathematical activities, it is necessary to go back and forth between process and object conceptions of a mathematical entity.

All of these conceptions of cosets together with properties that the individual understands are organized in what we call the individual's *schema* for cosets. A schema will be a collection of actions, processes, objects and other schemas, together with their relationships, that the individual understands in connection with cosets. This collection will be coherent in the sense that the individual will have some means (explicit or implicit), perhaps the formal definition, of determining, for any phenomenon encountered, what relationship it has to her or his conception of cosets.

Instructional treatment

In Figure 1, the arrow going from the theory to the instruction represents the specific mental constructions which the theoretical analysis proposes to the design of instruction. This design focuses on getting the students to make these mental constructions with knowledge of the mathematics involved as an expected by-product.

Several specific pedagogical strategies are used to help the students make the desired mental constructions. The most important of these are having students write computer code to implement mathematical concepts, and cooperative learning. In addition, there is an attempt to have the students engage in active learning, to figure things out on their own rather than rely on the instructor for explanations and information. This results in less lecturing than in a traditional class. Inputs from the instructor are designed to help students make the mental constructions. When the students have had an ample opportunity to make these constructions, the instructor may explain various concepts and methods to a small group of the students or to the whole class. These explanations are, again, based on the mental constructions the students are expected to have made.

The instructional treatment is organized in what is called the *ACE Teaching Cycle* of Activities to be done on the computer, Classroom discussions, and Exercises to be done with pencil and paper. A typical class will meet one or two days each in a computer lab and two or three days in a classroom.

In the computer lab, the students write computer code and programs and use these constructs to perform mathematical tasks. The computer activities are specifically designed with the mental constructions in mind. For example, to help the students construct an action understanding of cosets, they might be asked to write code such as:

```
G := {0..19};
op := |x,y -> (x + y) mod 20;
H := {0,4,8,12,16};
K := {3,7,11,15,19};
```

where the first two lines construct the additive group of integers mod 20, the next line forms a certain subgroup H and the last line gives the coset $K = 3 + H$.

For a process conception, the student might write,

```
G := {[a,b,c,d] :  a,b,c,d in {1..4} | #{a,b,c,d} = 4};
op := |p,q -> [p(q(i)) :  i in [1..4]]|;
H := {[1,2,3,4], [2,1,4,3], [3,4,1,2], [4,3,2,1]};
K := {[2,3,1,4] .op p :  p in G};
{p .op q :  p in H, q in K};
```

where this time the code constructs the group of permutations of four numbers with composition as the operation, the subgroup H which is isomorphic to the Klein 4-group, a coset K of H and the coset product HK.

Finally, to help the students construct an object conception of cosets, they might be asked to write a program that will accept any two cosets and return their product. It would look like,

```
CP := func(C1, C2);
            return {x .op y | x in C1, y in C2};
      end;
```

where it is assumed that a group operation op has been defined. This program can then be applied by the student to specific examples and used to investigate properties of the coset product, such as commutativity.

For more information on this way of using computers, see Dubinsky (1995).

In the classroom sessions, the students are given specific mathematical tasks to perform, based on the mental constructions they have made in the computer lab. For example, they might be asked to make a general statement about the number of elements in a coset or the intersection of two cosets, based on the examples they have worked with on the computer. Then the class could move to a proof of various properties that the students have observed empirically. In addition to working on these tasks, from time to time the students will listen to explanations (or brief lectures) by the instructor, who must decide when to let the students try to figure out something on their own and when they are ready to hear an explanation. This interplay between discovery and explanation is where the teacher has the greatest opportunity to control the pace of the course and apply her or his pedagogical creativity.

Finally, exercises are assigned to do as homeowrk. These are fairly traditional drill and practice as well as problems that require deeper thought. It is important to note that, unlike traditional instruction, the number of illustrative examples is minimized until the students have had ample opportunity to construct understandings of the mathematics involved. The reinforcement that comes from practice is an important part of learning, but it should not take place until the possibility of reinforcing misconceptions is reduced as much as possible.

All of the students' work in the course in the computer lab, in class, on the exercises and even some of the examinations, is done in cooperative groups which are established at the beginning of the course and not changed thereafter. I will say a little more about cooperative learning in the next section.

Data collection/analysis

During the instructional treatment and after it has been completed, data can be collected regarding the mental constructions the students may have made, the mathematics they have or have not learned, the attitudes towards mathematics they may have developed and how they have fared in the continuation of their studies in mathematics. A combination of qualitative and quantitative instruments are used in gathering this data.

In analyzing the data for mental constructions, the theoretical analysis serves as a guide for what to look for. That is, the question asked of the data is: to what extent did the students appear to make the mental constructions predicted by the theoretical analysis or what other constructions did they seem to make. This is done by making very fine grain comparisions between students who had difficulty with various very specific points and those that did not. The investigator looks in the data for explanations of the differences in terms of specfic actions, processes, objects, and/or schemas that were or were not constructed. At the same time, the data is used to evaluate the theoretical analysis and change it where appropriate. It is even possible that the data will suggest revisions to the overall theory. This two-way relationship between the data and the theoretical analysis is indicated by the two-headed arrow in Figure 1.

Relatively traditional instruments are generally used to determine the mathematics the students have learned and compare this with students who have taken courses in the same subject which did not use the pedagogical strategies described here.

Surveys and interviews are used to determine student attitudes and these are also compared with students whose courses used other pedagogical approaches.

Finally, surveys, classroom records and records from the registrars are used to obtain information about mathematics courses students subsequently took. A determination is made of how many courses they took and how they performed in those courses. Again, comparisons are made between students who experienced the pedagogy described here and those who did not.

If there are any revisions to the theoretical analysis arising out of a study, then the cycle of theoretical analysis, design and implementation of instruction, gathering and analyzing data is repeated, with an expectation of improved results.

A summary of the results of 14 studies conducted according to this framework is given in Clark et al (in preparation).

One approach to using cooperative learning

In this section, I would like to discuss in a little more detail one aspect of the pedagogical strategy used in our approach. Cooperative Learning appears to be a kind of pedagogy with two important features: it is difficult to get it to work properly, but when it does, it seems to be extremely successful in terms of student learning and their attitudes about mathematics. More information about this and other approaches to Cooperative Learning can be found in Rogers et al (in review) and more details about our particular way of using this method are given in Hegelgans et al (1995). Here I will discuss briefly the various ways that cooperative learning has been used and indicate some issues of concern, sketch some details of implementation for our use of cooperative learning, and point out some areas of resistance that are found in students as well as other faculty.

Ways of using Cooperative Learning and some issues of concern

The phrase *cooperative learning* has gained great currency in the last few years and, as generally happens in such cases, it comes to mean different things to different people. Following is a list of activities that have been referred to as cooperative learning.

- The instructor occasionally suggests to the students that they get together to work on the homework for the course.
- In class the instructor gives a problem and asks the students to work on it together in teams.
- The members of the class are organized in teams in some manner (e.g.,counting off, self-selection) to work on something specific (in class, project, etc.), using some particular strategy (jig-saw, assigned roles, etc.)
- Every few weeks, the class is organized in teams and some of the work in the course is done in those teams.
- At the beginning of the semester, teams of students are formed and it is expected that the students do all work in the course — homework, class discussions, and at least some of the exams — in their teams.

The list is in increasing order of the extent of use of cooperative learning and, it turns out, this is also increasing order of the difficulty of getting it to work. Nevertheless, although each of these strategies has value, my experience has been that the first two will not make much difference in the learning that takes place, the next two can be very effective and it is with the last that the full power of cooperative learning as a pedagogical strategy can be seen. Of course, no one should be surprised that the most effective pedagogy is also the hardest to implement.

The use of cooperative learning raises a number of issues and here is a list of the common questions that have been asked about it.

- How does cooperative learning fit in the overall class structure?
- How are teams formed and what is their structure?
- How do you test and grade students in a cooperative learning situation?

- What are the ways in which a team can function and how can that be influenced?
- What are some concerns of students?
- What are some concerns of faculty?
- What are effects on students attitudes about mathematics?
- Do students learn better?

I will try to deal with some of these issues in the remainder of this paper. In doing so, I will focus on the approach to cooperative learning that I have used. It is in the category of the last of the five items in the above list of activities using cooperative learning. As it is part of the *ACE Teaching Cycle* described above in the paragraph on Instructional Treatment, it relates to the first of this list of issues.

The following list describes the main considerations in forming and managing the teams which deals with the second issue in the above list.

- Groups of size three or four are permanent for at least one semester.
- There is an attempt to develop an espirit de corps within each group. We push the idea that every member of the group is responsible for the learning of all members of the group.
- Emphasis is placed on intra group interactions involving communication and explanations among group members, alternative appproaches to solving problems, correction of errors, and reflection on mathematical procedures.
- Most of the work in the course is done by the students in their groups.
- A significant amount of the assessment of students is done as a group.

For more information about this approach that has been and is being used by a large number of college faculty and about the details of implementation described in the net paragraph, see Hagelgans et al (1995).

Some implementation details

In this paragraph I will describe briefly the formation and structure of groups and the scheme for determining a student's grade in the course. This relates to the second and third items in the above list of questions that have been raised about cooperative learning.

When the course begins, students are given an opportunity to work in temporary groups of their own choosing in order to get some feel for who are the people with whom they can work well together. Some time in the first week or so, the students are asked to fill out a questionnaire about their mathematics background and ability, their experience with and attitude towards computers, their overall academic background and strength, and their place of residence. They are also given the opportunity to list individuals with whom they would like to be in a group and any with whom they would not. This questionnaire is used to form groups that are heterogeneous with respect to ability and background. An attempt is made to have people who live close together in the same group. Obviously, this is much easier at a school in a small town where most of the students live in a dorm, than at a school in a large city where many students live at home.

As for assessments, the basic idea is to get a balance between measuring performance of the group and measuring individual performance. Altogether there will be, typically, seven grade items. Usually there are two sets of homework assignments weekly, one based on the computer activities and one based on paper and pencil exercises. These are turned in as a group and the average scores on each provide two grade items. In general, every member of the group gets the same grade except that it is required that assignment submissions be signed by each member of the group indicating their full participation in doing the assignment. Occasionally, one member of the group will not do this and her or his grade is reduced.

There are three examinations in the course and these provide four grade items for each student. The first exam, taken about one-third of the way through the course, is a group exam and each student in the group receives the same grade. The second exam, given about two-thirds of the way through the course is taken individually, but each student receives two grades: their own score and the average of the scores of the members of their group. The final exam is taken individually and students only receive their own grade on the exam.

The seventh grade item is a measure of the student's participation in the classroom discussions. This is a group score and it is mainly used to resolve borderline cases. The course grade is largely determined by the first six assessment items. A rough average is taken, with the usual modifications due to considerations of improvement and special circumstances related to individual students. Students whose individual scores are all significantly higher than their group scores will be given special consideration and students whose individual scores are all significantly lower than their group scores may be penalized in their final grade.

Making it work

As I indicated in the beginning of this article, cooperative learning is one of the hardest pedagogical strategies to implement, but if one can manage to get it working, then it becomes one of the most powerful methods for helping students learn. The sources of difficulty lie in what mode of operation the students choose for their groups, the resistance of the students to working together and the resistance of other faculty to the use of this pedagogical strategy, especially with respect to group exams and group grades.

The mode of operation of a group in doing homework and other tasks can range from the entire group working together on every problem to dividing the problems up with each student working individually on her or his portion. In between there can be mixtures in which the group does some problems together and some individually. In general, there will be more learning when the group works together, but this takes time and practical considerations may force a group to work individually on some problems. The instructor can influence groups to work together more by meeting with them as a group, helping them work on problems of group interaction and trying to convince them to work as a group on problems as much as possible. Where there is a certain amount of dividing up the work, a good compromise is for the group to come together after group members have worked separately and share with the group what they have learned in their individual work.

By the time they have reached college and perhaps spent some time there, most students have worked in a rigid, competitive system in which the entire atmosphere is focused on getting the best possible grades. It is natural that students will be wary of a new approach and they can find it difficult to work together in replacing competition with cooperation. Students will complain that they cannot find time to meet in their groups outside of class. They will insist that they must be shown how to do every problem before being asked to do something simliar in class, for homework and on an exam. Often they will find that deep learning is a lot of work and takes a great deal of time. They will worry that their grades in other courses will suffer. Because the functioning of the course puts so much emphasis on the students taking responsibility for their own learning and becoming active learners, they will worry—largely because they have no experience with such a pedagogical approach—that they will not be sufficiently prepared for subsequent courses. These concerns will often lead to a weakened effort in the course and/or giving low evaluations of the course and the teacher.

The instructor must be sensitive to these concerns and try to convince the student through persuasive arguments, personal enthusiasm, and the records that are beginning to be established. As will be discussed below, there is mounting evidence showing that, for the most part, students' fears are not justified and the instructor can share some of this data with them. As the course progresses, students will begin to see that they are learning more and their efforts are producing results without hurting them in other courses. Seeing that working together is very much in their interests is another kind of learning that can take place in a course that uses cooperative learning effectively. After this method has been used at a school for some time, the long range positive effects of the experience (for which some evidence will be described below) for students who have taken courses using this strategy in the past will become part of the information that young students always receive from their predecessors.

Concerns and resistance by other faculty is a different matter. There are a number of misconceptions about cooperative learning and reformed pedagogy in general. Many faculty are convinced that these new approaches require students to reinvent all mathematical concepts on their own, that the course will not cover enough material, that students don't want to cooperate with each other, and that cheating will increase. There is a resistance to increasing the amount of time they spend preparing and an anxiety about reducing lecture time and figuring out what kind of activities to replace lecturing with. They worry, quite understandably, about the effect on their career if their student evaluations become lower and they are not convinced that these new methods lead to significant improvements in student learning.

It is not the case that students are required to discover everything on their own. The pedagogical strategy is designed to give students an *opportunity* to make their own discoveries. The instructor can decide how much time they spend doing this and when to stop investigation in order to have explanations given either by students who have succeeded or by the instructor. The point is that an explanation is much more effective for someone who has tried to figure things out for her- or himself. By deciding how much of an opportunity for discovery the students will have, the instructor can control the time it takes to cover a topic. It is true that real learning can take a long time and the total number of topics in a course using strategies we are discussing can be less than in traditional courses. The differences, however, are not large and can be offset by the deeper level of understanding the students have for the material that has been considered in the course and the ability many students develop in such courses to work things out on their own when necessary, even if they have not been covered in the course.

It is also true that teaching in ways described in this article can require more time and effort by the teacher, at least at first. It is hoped that as the evidence for the effectiveness of such approaches grows, administrations will respond by finding ways to reward teaching that is more effective and provide for the extra efforts that may be required. In any case, as more and more courses of this kind are developed, materials will be available and teachers will not have to produce it on their own.

Of course, everything rests on the question of the effectiveness of these new pedagogical approaches. The evidence for that is mounting and we will discuss it in the next section.

Effects on student learning and on their attitudes

As we have indicated, our pedagogical strategies include not only cooperative learning, but also having students perform various tasks on the computer and reducing lecturing in favor of students working on paper-and-pencil tasks. In gathering data on our approach, we have not isolated these components in order to study their individual effects. The reason for this is that we are convinced that all of these approaches will lead to a better learning experience for students and so, to the extent possible, we insist on using all of them in every course.

As mentioned earlier, the effects of our methods have been the object of 14 studies that are summarized in Clark et al (in preparation). The mathematical topics considered here include topics in calclulus such as students' understanding of the chain rule, limits, infinite sequences of numbers, the definite integral and their graphical understanding of the derivative of a function. It also includes topics in abstract algebra such as binary operations, groups, subgroups, cosets, normality, and quotient groups. Finally, several topics in discrete mathematics are studied such as functions, quantification, mathematical induction, permutations and symmetry.

The data includes both comparisons of students who have taken courses based on our methods with students who have taken courses based on traditional pedagogy and non-comparative data of students experiencing our pedagogy. In the former case, we found that in most of our assessment items the students who took our courses outperformed the others substantially and on the other items there was little or no difference. The non-comparative data indicated that the student performance was in general better than one would expect from such students in these courses. The data also strongly supports the general theory used in this approach as a model for describing the learning process.

Effects on student attitudes

In addition to data about learning the mathematics in these courses, we also gathered data about the students' attitudes towards the mathematics studied and the methodology that was used. There were three major studies: long term effects of the calculus sequence at a large mid-western university; attitudes towards abstract algebra as a result of a single course in this subject at that university; and attitudes towards mathematics in general as a result of taking courses in calculus, abstract algebra and discrete mathematics at a large city university in the southeast. I would like to give some details on the results of these studies.

The study of long term effects of calculus courses looked at various records over a 4 year period of 4636 students, 205 from a sequence of calculus courses using our approach which we will call C^4L, and 4431 from the traditional calculus sequence which we will call TRAD. Following is a summary of the results. The actual numbers are given in Clark et al (in preparation).

- The C^4L students were more likely to finish the calculus sequence than were the TRAD students.
- The C^4L students earned higher grades in calculus courses than the TRAD students.
- C^4L students took more non-calculus mathematics courses than did the TRAD students.
- The C^4L students did about as well in mathematics courses beyond the calculus programs as did the TRAD students.

Similar data was gathered about students who had taken courses in pre-calculus, calculus, abstract algebra and discrete mathematics over a three year period at the city univesity. The results were the same.

The differences with respect to abstract algebra attitudes were striking. It was even difficult to get students who had taken the traditional course to come for interviews, although they were paid. Many said that their experiences were so unpleasant that it would be upsetting for them to spend any more time discussing this subject. One student who had taken the traditional course walked out of the interview as soon as he realized it would not be an opportunity for him to register an official complaint about the course.

A total of 33 students were interviewed, 11 from an abstract algebra course using our methods and 22 from traditional abstract algebra course. The interview was not designed to look at attitudes, but rather at learning. The reactions of students reported here were therefore mainly spontaneous.

- 8 of the 11 students from our course and 7 of the 22 students from the traditional course made generally positive comments.
- None of the 11 students from our course and 4 of the 22 students from the traditional course received an A or B in the course and made only negative comments.
- 3 of the 11 students from our course and 14 of the 22 students from the traditional course commented on how difficult the course was.
- 8 of the 11 students from our course and none of the 22 students from the traditional course said they had learned a great deal.

For the study at the city university, we mailed questionnaires to about 200 students 1-2 years after they had taken courses in precalculus, calculus, discrete mathematics, and abstract algebra. We received responses from 90 students, 44 from courses using our methods (EXP) and 46 from traditional courses (TRAD).

Students were asked to express the extent of their agreement or disagreement with various statements on a scale of 1 (strongly disagree) to 5 (strongly agree.)

Following are the statements with the average response from the two groups.

- I enjoy doing math problems. EXP - 4.1, TRAD - 3.7
- In mathematics, an answer is either right or wrong. EXP - 3.1, TRAD - 3.5
- Mathematics requires much more memorization than understanding. EXP - 1.7, TRAD - 2.2
- In the long run, mathematics will help me. EXP - 4.4, TRAD - 4.3
- In order to understand math, I need to know more theory. EXP - 3.3, TRAD - 3.2
- I learn math through examples. EXP - 4.54, TRAD - 4.51
- Guessing is OK to use in solving math problems. EXP - 2.7, TRAD - 2.3
- Calculus requires more thinking than memorization. EXP - 3.9, TRAD - 3.4
- I can apply calculus in some other courses. EXP - 3.8, TRAD - 3.4
- A good math teacher shows how to do problems that will be on the test. EXP - 3.9, TRAD - 4.1
- A good math teacher shows different ways to look at a problem. EXP - 4.5, TRAD - 4.2

With the possible exception of the comment about examples, every response from the EXP students is preferable to the corresponding response from the TRAD students.

Conclusions

The numbers we have comparing students who expeienced our pedagogical approach show differences that are occasonally large, but generally small although often satisfying tests of statistical significance. Moreover, where there are differences they almost always in favor of the students who experienced our pedagogical approach. The non-comparative data indicates a degree of learning mathematics that is more than statisfactory, given the nature of the courses and of the students who took them.

The data we have gathered is not inconsistent with the assertion that, in our approach, calculus was a pump not a filter; students seemed to learn more, took more math courses and their grades in other courses, both in mathematics and overall, were not adversely affected; and the attitudes of students towards these courses and in mathematics in general was gratifying, especially some time after they took the courses using our approach.

This last point, together with the fact that student evaluations can sometimes be lower for courses using the pedagogy described in this paper. This negative effect occurs just at the end of course but disappears and may even reverse some time later. Unfortunately, administrative decisions about faculty that try to take into account teaching effectiveness generally use the student evaluations and make no attempt estimate this characteristics of a faculty member in the long run. As a result, it may be happening that some faculty will tend to avoid using what may well be a more effective method of teaching because of the short-term dangers to their careers. It is a challenge to the administrators of our colleges to find ways to differentiate between short-term, negative but fleeting student reactions to a course or a faculty member and take into account what may be more positive results that are manifested only in the long run.

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