SOME KINDS OF COMPUTERS FOR SOME KINDS OF MATHEMATICAL LEARNING - A REPLY TO KOBLITZ

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INTRODUCTORY REMARKS

It is becoming increasingly urgent to examine the computer's role in mathematical teaching and learning - no less important than the discussion of the computer's place in mathematics itself. The initial flush of exuberance over "new technologies" is beginning to pass, and it is time for mathematicians to think carefully about the ways in which computers may or may not help students learn their subject, preferably on the basis of reasoned argument, well-informed knowledge of the research evidence, and dispassionate examination of the costs, benefits, and difficulties.

Unfortunately, Koblitz's contribution is neither dispassionate, nor well-informed. In this respect it fails to take forward the development of criteria for the acceptance or rejection of technology in mathematical teaching. More importantly, by subsuming under the guise of "the computer" any and all software, the whole gamut of learners, and (as we shall see) a surprising range of what might count as "mathematics", he fails to contribute seriously to the debate and - ironically - hands by default decisions on the computer's use to the computer companies and software giants which Koblitz - and ourselves - wish to disparage.

Koblitz begins by asking if computers are what schools in Third World countries need, or if resources could be spent in better ways. A fair question. Yet the principle of self determination, accepted in name, if not in action, by almost everyone, suggests that this is a question which each Third World country should decide for itself. It seems inappropriate for a U.S mathematician from a prestigious research university writing in a sophisticated mathematical publication to declare that it is "bizarre that the Tunisia conference adopted as an axiom that the introduction of computers should be a priority for elementary education in Africa."

It is true, of course, that any country deciding on how to allocate resources in, for example, education, should base this decision on a clear understanding of the merits and demerits of various calls on these resources, such as the introduction of computers. It is here that professionals can make a contribution by analyzing research as well as experience with learning and technology to help provide a knowledge base for these decisions. Indeed, after making a number of remarks about computers in the Third World, Koblitz devotes most of his article to a discussion of the value of computers in education. Unfortunately, this discussion replaces his initial impropriety with a view of how computers are used in education that is so narrow and exclusive as to border on distortion.

Following Koblitz' organization, we would like to begin with some remarks about his discussion of the economics of educational commodities, which come from the industrialized countries and are sold wherever the market can be found - in first, second, or n-th worlds. Then, we would like to consider Koblitz' arguments about the use of computers in education (in any i-th world) and try to point out that if one takes a somewhat broader view, it might be possible to perceive phenomena of which Koblitz seems unaware and which could help in reaching conclusions very different from the ones he proposes.

Our main theme is that there are ways in which a computer environment can be designed and used to help students learn mathematics that are very different from the approaches which Koblitz (quite properly, for the most part) rejects; moreover, there is reason to believe that these methods can be extremely effective. We hope that by addressing the most important subset of Koblitz's arguments, we will have thrown some light on the all important question of what might be done to increase the effective use of computers in education and decrease the ineffective use.

EDUCATIONAL USE OF COMPUTERS IN THE THIRD WORLD

Koblitz suggests that computers add "insult to injury" in the third world, and are primarily aimed at expanding the market base of the first world manufacturing industry. This point is essentially correct. Indeed, it is true in the first world as well: for example, the introduction of computers into UK schools in the nineteen eighties was handled by a government department - "The Department of Trade and Industry" - whose brief was to subsidise and expand the UK computer industry at the expense of educational benefit. It is also a phenomenon that is not restricted to technology. Textbook publishing is a billion dollar industry and everything (including sales of left over stock in third world countries) Koblitz says about technology is, and has been for some time, true in this industry as well. The only thing new is that textbook publishing is joined by technology publishing. Of course the owners and profit-takers won't change their priorities, they'll just retool.

But the conclusions to be drawn are not so clear. Take the example of medicine. The development of medicines by the huge multinational pharmaceutical corporations are designed for profit first, the alleviation of human suffering (a very poor) second. The exploitation of the third world market certainly adds insult to injury: take the case of marketing artificial milk products which is undeniably killing babies throughout the "undeveloped" world.

Should we stop the development of new medicines? Certainly not. Should we try to control this development, so that the primary goal is to alleviate human suffering rather than generate profit? Certainly. Should we be prepared to consider individual drugs carefully, subject them to scientific research, or should we subsume everything (penicillin, alcohol, tobacco, cocaine, aspirin) under the heading "drug" without distinguishing between them, and conclude that "drugs" are either "good" or "bad"? And finally, how much of our attention and resources should we put into the development of cures for our illnesses as opposed to looking at, and possibly changing, those aspects of our lifestyle which tend to cause some of our illnesses? Many cultures (e.g., Chinese, Native American) have medical traditions that have as much to do with how medicines are used and how they are integrated with the lifestyle as with the chemical nature of the medicine itself.

In other words, you have to think about medicines in relation to the social environment in which they are produced and used. In the same way, computers have to be thought about in relation to the total environment in which they are used. Computers are used both for missile guidance and micro-surgery. In education they are used both as expensive (and largely ineffective) child-minders, and as expressive and powerful tools. It is the culture which determines how the technology enters, not the technology itself. As one of us has written elsewhere, "Pieces of knowledge are appropriated (or not) depending upon pupils' own agendas, how they feel about their participation, teacher intervention, and above all, the setting in which the [computer] activities are undertaken." (Hoyles and Noss, 1992.)

RESOURCES AND CULTURES

Koblitz attacks computers as the epitome of "popular culture", which is based on passive doses of "entertainment" and, in the educational domain, entails a "dumbing down" of intellectual content. Of course. Our entire modern economic and social life is based on the commodification of essential physical, emotional, and intellectual life: to cast people in the role of consumers rather than rounded human beings (see Noss, 1994 for an elaboration of this argument). But this is true for all technology: indeed, it is the central role which technology is called upon to play in modern capitalist economies.

But here, as in many instances, Koblitz has a simplistic view of the world, one in which there is good and evil, in which darkness must be vanquished by the forces of light. Life is not so simple. Contained within the undoubtedly negative consequences of technological passivity are seeds of contradictory and counterposing forces. Once again, it is not just the nature of a piece of technology, but how it is used. The same technology which, as Koblitz rightly observes, can be used to lull the unsuspecting child into a passive inability to "concentrate for long periods" can hold a child transfixed for hours; the machines which offer immediate gratification for minimal reading and writing skills, can also be used by young children to craft complex and meaningful pieces of writing, and to practice the new (for children) skills of editing, and redrafting; the same computer that runs software in which sophisticated mathematics problems are solved simply by pushing the right buttons, can also support systems that students use to construct mathematical tools on the computer and simultaneously construct mathematical concepts in their minds.

We can no more claim that "computers" are the source of a passive, popular culture, than we can blame "medicine" for encouraging Indian mothers to reject breast-feeding. Once again, one of our biggest disagreements with Koblitz is that he bases his argument on one way of using computers - admittedly a widespread way - and ignores other uses about which his complaints are not justified.

We are also not much enamored of Koblitz' alternative which is, apparently, that a mathematician gets up a set of notes for a course and if the notes are good enough, calculus will be fixed. Where have we heard that before? It is simply not true that if we continue to do things in the same old way, but do them better, everything will be alright. There are many incredibly talented and dedicated practitioners of traditional pedagogies and still, even in the industrialized countries, children are neither learning nor liking mathematics. It would be a sorry shame if, once again, the industrialized world exported its failures to the developing world.

Koblitz asserts that computers are a "drain on resources". We are certainly opposed to any attempt (and such attempts are underway) to replace teachers by computers. Computers alone can not make good teachers and we are not encouraged by attempts to get computers to do the same things that teachers do (see Noss, 1995, for evidence of this assertion in the context of undergraduate mathematics teaching). We agree with Koblitz that computers are a drain on resources if they are used as an excuse to fire teachers. Teachers, books, desks, roofs over classrooms, warmth, and running water are all greater priorities. But we argue that there are ways in which an environment that includes all of these and also includes appropriate uses of computers, can lead to situations in which more students have richer educational experiences.

In addition to the potential value of the computer as part of a rich educational environment, it can also happen that the existence of computers creates exigencies that lead to more of what Koblitz is calling for. For example, the huge expenditures by the National Science Foundation in the U.S. in support of calculus reform in particular and teacher enhancement in general have been, at least in part, driven by the appearance of computers and the need to learn how to incorporate them into our educational practice. In the UK, the appearance of pocket computers is at last forcing a rethinking of the higher levels of the mathematics curriculum which have focused unchanged for centuries on the acquisition of skills and techniques which are rapidly becoming anachronisms. The computer in all its forms is catalysing debate and reform in what it means to teach and learn mathematics.

THE CHARGE OF ANTI-INTELLECTUALISM

Koblitz argues that computers foster an anti-intellectual appeal. This argument is essentially the same as his earlier point concerning passivity and consumerism: it is hardly surprising that this most "modern" of technologies incorporates the "Golly-Gee-Whiz" approach that Koblitz criticises. "Most software is based on immediate gratification, and does not encourage sustained mental effort" says Koblitz. In so far as most software does exactly this, we reject it also. But a small (yet nonetheless substantial) fraction does precisely the reverse; indeed it does not "do" very much at all. The word processor with which we are writing, or the high level programming language in which we are expressing mathematical ideas do not "do", "encourage", or "gratify" in any way at all. Each of these is a medium of expression, like a piano, or a pen and none of them are, in themselves, either creative or uncreative. To argue that computers, pianos or pens offer "little opportunity to be creative" is to miss the point: creativity is a state of mind, a social response to situations generated by people interacting with each other; people doing things with whatever cultural tools they have at their disposal. To lay the blame for lack of creativity on "the computer" is like lambasting the violin for the boredom of listening to some particular musical composition.

In the case of learning mathematics, we could not agree more with Koblitz that "teaching by demo" is bad pedagogy. But we cannot avoid the observation that amongst the worst examples of this pedagogical strategy we must include having students listen to lectures, follow worked out problems in a text or read notes written by a mathematician who is not paying attention to how learning actually occurs.

COMPUTERS AND PEDAGOGY

Now we come to the meat of Koblitz' paper. He claims that computers generate "bad pedagogy". He attacks the "grandiose claims" of, among others, Seymour Papert, and bases his evidence on several references. Some of these (e.g. Cuffaro, Sloan, Cuban) are ten-year old expressions of opinion, belonging to an interesting but bygone era when it was fashionable to discuss whether computers were as good as paintbrushes for artistic expression, or whether computer-sounds were as pleasant as those generated by a Stradivarius. Such discussions were appealing to some, and many of the proponents on both sides expressed their opinions in suitably polemical terms: but "evidence" they are not, and it is somewhat disingenuous of Koblitz to cite them as such.

We have two points to make regarding computers and pedagogy. One is that, as we have indicated above, there are a multitude of ways in which computers can be used in education. These include microworlds in which mathematical phenomena are an integral part of the learner's environment; computer algebra systems that students use to explore mathematical ideas and solve sophisticated problems; mathematical programming languages used to construct mathematical concepts on the computer; graphing facilities which allow the learner to see complex phenomena; multiple representation software in which a student can see the effect that changing a feature of one representation has on the others; playful and experimental "games" in which learners try to accomplish entertaining tasks; and spreadsheets in which the structure of the software provides a powerful way to model some mathematical situations.

We do not assert that everything done with all of these pedagogical tools is "good", nor are we agnostic as to their relative merits (neither do we suggest they are well-ordered). But we do claim that there is a lot more to this multitude of software than is contained in Koblitz' simplistic report - and much of it, used in appropriate ways can make a significant difference in how much mathematics is learned by how many people.

This improvement in learning is the second point we wish to make in response to Koblitz' rejection of "grandiose claims". True, there are no grandiose claims that can be justified. But there is a growing literature that explains in detail the uses of computers that do help and provides evidence of the effectiveness of these pedagogical strategies. There is not room in this brief essay to describe this literature, but we can mention some examples. For effects of having students write programs to learn mathematical induction see Dubinsky, 1989; to learn predicate calculus see Dubinsky, in press; and to learn functions see Breidenbach et al, 1992. The contributions in Hoyles and Noss (1992) testify to the breadth of the encounter between computer programming in Logo, and a range of mathematical topics including three-dimensional geometry, group theory and dynamical systems. For an overview of experiments showing both success and failure of ways of using calculators to help students learn various mathematical concepts, see Dunham, 1993 and for a description of how spreadsheets can be used in mathematics education, see Smith, 1992. And the diverse contributions to diSessa, Hoyles and Noss (1995) amply illustrates the wide range of mathematical ideas which the computer (e.g. programming languages such as Logo and Boxer, dynamic geometry tools such as "Cabri Geometry" and the trusty spreadsheet) offer new insights for learners and teachers alike.

We feel that this literature is quite different from Koblitz' choice of the holy grail of research, the paper which has become a necessary and sufficient condition for any argument which purports to show that computers have no (or, indeed, a detrimental) effect on mathematical learning. This paper is by Roy Pea and Midian Kurland, two researchers from Bank Street college who, in the early nineteen-eighties, undertook a series of studies of the computer programming language Logo, and its "effects" on learning mathematics. It is regrettable that Koblitz joins the substantial number of writers who trawl out Pea and Kurland as evidence without comment. For the benefit of readers who may be unfamiliar with Pea and Kurland's research, we outline in the following three paragraphs what they "found".

Pea and Kurland mounted a series of studies in the early nineteen-eighties (a typical and oft-quoted example is Pea and Kurland, 1984). Their intended aim was to explore the "effects" of young children learning to program a computer in the programming language Logo. As psychologists, not mathematicians, they were interested in "transfer" of "human cognition". In a typical experiment they allocated (not randomly) a sample of twenty four children from a private school half of whom learned Logo, and half who did not (we are not told what these children did instead). At the end of a four-month period, the children were asked to devise a plan to carry out 6 classroom chores on a transparent plexiglass map of a fictitious classroom. Success on this task (as measured by a fairly arcane set of criteria) was found to be not significantly different between the Logo and "control" groups.

Now there is much to be said about this experiment, concerning misplaced objectives, faulty statistics, and inadequate methodology (see Hoyles and Noss, in press). But we will restrict ourselves to the most obvious and relevant point for Intelligencer readers: this "finding" tells us nothing about the learning of mathematics. It does not tell us, as Koblitz argues, that "all the claims made about the beneficial educational effects of learning to program are not only inflated, but probably incorrect". Neither does it show that "there is not even support for the ... notion that learning to program aids children's mathematical thinking". It is simply an irrelevance. Worse, an irrelevance based on a finding of "no significant difference": surely it is rather difficult to assess the value of a counterexample unless one knows what it's a counterexample to?

But perhaps most telling is the complete failure of Pea and Kurland to document just what the children did with Logo. Were they given specific activities? If so, what kind? If not, what were they given instead? Did the activities focus on the idea of planning which would subsequently be tested? We simply do not know. Pea and Kurland showed that young children who had been given a copy of Logo were no better at solving a cute puzzle involving pushing in chairs, and cleaning tables, than those who had not. So what?

The reality of Pea and Kurland's research, and the contrast with the way it is mobilised by Koblitz, points to the bottom line on the literature in this field. There are many reports, some of which may be considered as evidence and some not; but research needs to be carefully weighed against our common criteria - that of making the learning of mathematics more effective, enjoyable, and successful - not by counterposing "good" with "bad", or passing as "findings" reports of studies which neither know nor care about the subject we are trying to address.

WHAT ARE WE TRYING TO ACHIEVE?

We find the following curious sentence in Koblitz' article. "If the best computers in the world are unable to translate from French into English, then they certainly cannot help my calculus students do what is the main point of the course: translating word problems into mathematics".

This revealing quote (p.6) allows us to address two further questions simultaneously. First, the nature of what mathematics we would like students to know, and second, the evidence which Koblitz manifestly fails to mobilise concerning the learning of this, or any other, mathematics.

First, mathematics itself. We share Koblitz' goal that students should learn to translate word problems, although we think of it as using mathematical structures to model situations described in word problems. It is a kind of superficial global thinking to insist that the computer be capable of doing what we ultimately wish the student to accomplish. It would be like rejecting a piano because it can't sing. Jut as the piano has a critical role in the performance of an oratorio, the computer can be an essential ingredient in certain kinds of learning experiences.

But there is a deeper issue. If Koblitz really believes that the "main point" of learning calculus is the translation of word problems in the way that he could imagine a computer doing, then he is entitled to this view, but we cannot share it. Our preference is to help students learn to respond to a situation by mathematizing it, by constructing functions and relationships which can be used to make sense of it, and by using tools such as calculus to answer questions about it. But this is a very different matter from mindless translation of the sort that computers might (but, in general, cannot) do. It may be that we are not at odds with Koblitz on this substantive point, except that he blames the computer for a failure to think through an appropriate pedagogy. Let's agree that translation (in the sense of modeling) is important: why can't we use appropriate computational tools to help us, and reject those which are inappropriate? Certainly the work we have referred to suggests that appropriate uses of these tools do exist.

Turning to computers themselves, Koblitz offers the following banal observation: "What children need in order to become mathematically literate citizens in the computer age is not early exposure to manipulating a keyboard, but rather wide-ranging experience working in a creative and exciting way with algorithms, problem-solving techniques and logical modes of thought."

There are two surprises in this statement. The first is that Koblitz should think that anybody would think otherwise, other than the advertising agents of the computer companies who he (and we equally) despise. The second, is the word "rather". Why are the two counterposed? Why should creative and exciting work for children be restricted to some technologies but not others? Are pens and paper acceptable technologies? Presumably. Light pens and word processors presumably not. Dynamic geometry tools and programming languages, definitely not. We ask Koblitz the following question: where does he draw the technological divide? Does the inclusion of a chip rule out the use of the tool? And if so, how is it that the tools used by the most creative professionals (including creators and users of mathematics) should be closed to children?

Furthermore, we wonder about Koblitz' suggestion that manipulating a keyboard is the only, or even the main way of interacting with a computer. Is this one more case in which he focuses on the least exciting aspect of his straw man and pretends that the rest does not exist?

CASH, COMPUTERS, AND CHOICES

Koblitz says that "money corrupts". He reiterates the negative effect of corporate profit on educational benefit. We concur. He ridicules "believers" who say that "graphics calculators have changed my life". We take his point. But we cannot understand his view of himself as a "nonbeliever". Our (horror: electronic) thesaurus says Belief: creed, conviction, faith, tenet, doctrine. It is a shame that Koblitz reduces discussion of the learning of mathematics to the level of fundamentalist religious prejudice: we prefer reasoned argument based on research evidence.

In any case, if Koblitz is really concerned about the corruptive effect of cash, then he should go after the system in which the only way that educational materials can be produced and disseminated on a broad scale is if someone makes a profit. In such a system, it is natural that profit makers want more profit and so there is created a conflict between what seems to make good sense educationally and what will sell. Many calculus reformers in the U.S. are just now having direct experience of how difficult this makes it to maintain innovations in the materials they produce. This is a very big problem, but Koblitz is wrong in suggesting that it has to do with computers. It goes much deeper.

In an understandable but misguided attempt to resolve this dilemma, Koblitz makes his final point: "low-tech is better". He offers us four examples: i. a nice little code-breaking example; ii. an unexceptional exercise in combinatorics; an interesting example of data analysis, based on economic inequalities in "a certain Third World country"; and finally a "real world" calculus word problem. This last question, by the way, is based on the continuously compounded interest payable on a car loan. We note in passing that the technology of the internal combustion engine, the business practice of banks, and the role of the car in consumerism and advertising culture are, it seems, on the right side of Koblitz' technological divide.

Two final questions on "real world" problems. Whose real world does Koblitz have in mind, and can he cite a single study that shows that using this kind of example has any positive effect on student learning? We wouldn't even dare to ask for such information about the effectiveness of the "low-tech approach using some applications-oriented lecture notes that [Koblitz] had written". Does Koblitz really think that such an approach has not been tried by a multitude of mathematics teachers, including most of those who, after trying such simplistic approaches and looking at the results, have come to the view that much more serious thinking about learning is needed. There are some of us, mathematicians and educators, who feel that we are hardly in a position to be complacent, and that we cannot afford to reject out of hand new technologies that might - just might - help us in our task.

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