

## A Radical Constructivist does CL

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I consider myself to be a radical constructivist. Presumably this implies that I know what radical constructivism means. The term was coined by Ernst von Glasersfeld, but the idea goes back, at least, to Jean Piaget. My understanding of this philosophical position is that there is no external reality, or at least a human being can never know that there is, but rather each person constructs reality for her- or himself. This notion of reality applies both to ideas in our minds and to what we refer to as physical reality, or the "real world".

I suppose it is the inclusion of physical reality among the phenomena we construct that leads to the adjective "radical". For the purpose of discussing such matters, I think it is reasonable to define physical reality as those phenomena to which we have access through our senses such as books and chairs and rocks; whereas we might include all other phenomena such as concepts of love, as well as mathematical concepts, for example, function, in what we might call mental reality.

So we can summarize radical constructivism as a focus on construction of our understanding of all phenomena that we seem to experience, including both our interaction with physical objects and with the ideas of ourselves and of others. Of course this construction that an individual makes is not done in complete isolation. No radical constructivist would deny that if, for example, an apple should fall on our head, then something has happened which is, at least in part, external to us (an apple fell). The point is that we cannot know anything about this external happening. All we can know is that our head hurts, that we saw (and perhaps heard) something which we interpret to be an apple in motion, and that there is a fresh apple lying on the ground. The rest of our knowledge about this event is our interpretation. The same holds for mental reality. In thinking about a function, for example, we might be trying to make sense out of the relation between time and the position of our apple, and we might even construct a formula that gives this position for any given time within a certain range. But we talk with others about these constructions, before, during, and after we make them and what we say must be compatible with what others are saying. In other words, although I cannot really be sure that my understanding of the function that describes the motion of the apple is the same as yours, we can --- and do --- insist that our respective understandings fit with each other, that there should be some agreement between what the two of us say about this function. It might even be that my construction of my understanding of this function (which we express, for short, as my construction of this function) comes as a result, at least in part, of interactions with you (e.g., conversation, listening to you lecture, reading a book you wrote, etc.)

It is this social factor that leads to a relation between the construction of knowledge and cooperative learning (CL). Obviously, if interaction with others is part of the construction of knowledge then a learner should have as much interaction as possible with the instructor and with other learners. It is the goal of having a two-way interaction between teacher and student that has led me to change my teaching style by replacing the lecture in which I do all of the explanation by discussions in which both the student and I try to express the ideas we are constructing. Similarly, the goal of multi-directional interaction among students is one of the reasons I use cooperative learning in all of my teaching and curriculum development.

So, in my teaching of collegiate level mathematics courses, I put the students in groups at the beginning of the semester and they remain in the same group for the entire course. The reason for this is that it takes time for a group to develop as the members learn how to interact with each other in the most productive ways, and I don't want to change a group just as it becomes productive.

Usually, I am using computers and the study of each mathematical topic begins with the students, working in their groups in a computer lab, performing certain tasks on the computer designed to foster their making certain mental constructions which research and analysis suggests can lead to their constructing an understanding of the topic. Instruction on exactly how to perform the tasks is minimal, partly because this stimulates interaction with the teacher and within the group. After working on the lab, the students come to class where

I try to get them to reflect with their group on the experiences they had in the lab, and to work cooperatively to perform, with pencil and paper, other tasks, this time more closely related to standard mathematics problems and exercises.

The constructions of mathematical knowledge that students made in the lab and the classroom are reinforced through exercises they do for homework. Their knowledge is evaluated through examinations. All of their homework is done in their groups and each group submits a single assignment. About half of the testing is done in groups and half as individuals. I generally give three exams during the semester and a final. Usually I arrange matters so that the students have as much time as they need for the exams which are closed-book. The first exam is taken in their groups and every member of the group gets the same grade. The second exam is taken individually, but each student receives two grades. The first is the score of her or his exam and the second is the average of the scores of the members of the student's group. The third exam and the final are taken individually and the students get only their individual score.

As a constructivist, I cannot assert that this information tells about the actual knowledge that students have constructed, but only indicates the extent to which their knowledge fits with what mathematicians may be thinking about the topics on which the students are tested. Nevertheless, the results are used in assigning course grades. In our research we also use this data but only in conjunction with other instruments such as in-depth interviews.

I and others have been using this approach for over a decade and we have published papers with data indicating its effectiveness. Of course CL is not the only pedagogical strategy we are using. Our approach uses research and epistemological analyses of concepts, having students implement mathematical concepts on the computer and the minimization of lecturing in favor of small-group problem solving. As an almost moral stand, because we believe that all of these factors contribute to student learning and we want every course to be as good as it possibly can, we have never conducted experiments that try to isolate factors and determine their individual effectiveness. Therefore, we cannot assert that CL alone helps students learn. What the data does suggest is that CL, together with these other factors appears to be very effective in helping students learn collegiate level mathematics concepts.