The final is worth 30% of your final grade. Please work hard to prepare for it. The best way to prepare for the final is to read each section of the book we studied and go over all the exams and homework problems. If you want to get a good grade, make sure understand all the old exams problems and the following is a checklist.

Show all of your work in order to receive full credit.

(1) a) State the definition of the derivative (as a function) of a function \( f(x) \). Understand the difference of the definitions between \( f'(a) \) and \( f'(x) \).

b) Use the definition of derivative (as a limit) to find the function derivative, \( f'(x) \), of \( f(x) = \sqrt{x - 3} \).

(2) For each of the following, determine if the function is continuous at the given point. If it is not continuous at the point, classify the discontinuity.

(a) \( f(x) = \frac{x^2 - 2x - 3}{x^2 + 3x + 2}, a = -2 \)

(b) \( f(x) = \frac{x^2 - 3}{4 - x}, a = 1 \)

(c) \( f(x) = \frac{x^3 - 1}{x - 1} \)

(d) \( f(x) = |x| - 1 \)

(e) \( f(x) = \frac{1}{x} \)

(3) a) Find the equation of the tangent line to the curve \( f(x) = e^{x^2+2} \) at the point \((0, e^2)\).

b) Find the numerical value of \( \sum_{i=1}^{27} (1 - 2i)^2 \) (show the appropriate formula(s) you are using)

(5) Find \((f^{-1})'(3)\) if \( f(x) = \sqrt{x^3 + x^2 + x + 9} \)

(6) Let \( f(x) = \frac{7}{x^2 + 1}, f(x) = \frac{2x^2}{x^2 - 1}, f(x) = \ln(4 - x^2), f(x) = 2^x - 1, f(x) = \sin^{-1}x, \) and \( f(x) = \frac{x^2}{\sqrt{x+1}} \) separately,

(a) Find the domain of \( f \).
(b) Intercepts of \( f \).
(c) Asymptote(s)
(d) Determine intervals of increase and decrease
(e) At what value(s) of \( x \) do the local minimums and maximums occur?
(f) Determine the intervals on which \( f \) is concave up or concave down
(g) At values of \( x \) do the inflection points occur?
(h) Sketch the graph of \( f \) (include relevant points on the x-axis only).

(7a) What is the relationship between continuity and differentiability? That is, which implies the other?

(b) Give an example of a function for which the converse implication fails and explain why.

(8) Find \( y'' \) if \( x^2 - y^2 = 7 \) and simplify completely.

(9) Evaluate \( \int_{\frac{1}{2}}^{\frac{5}{2}} |4x - x^3| \, dx \) by appropriately splitting the integral.
(10) Calculate the following limits (you may need to use L'Hospital’s rule on some of them)

(a) \( \lim_{x \to 1} \frac{1}{(x - 1)^2} \)

(b) \( \lim_{x \to -\infty} \sin \frac{1}{x} \)

(c) \( \lim_{x \to -2} \frac{x^2 + 3x + 2}{x^2 - 1} \)

(d) \( \lim_{x \to 1} \frac{x^b - 1}{e^x - 1} \)

(e) \( \lim_{x \to \infty} \frac{\sin x}{x} \)

(f) \( \lim_{x \to \infty} \frac{\ln x}{\sqrt{x}} \)

(g) \( \lim_{x \to \pi} \frac{\sin x}{1 - \cos x} \)

(11) Evaluate the following integrals

(a) \( \int \frac{x^7 - x^4 + 6}{4x^2} \, dx \)

(b) \( \int (\frac{1}{1 + x^2} + \frac{2x}{1 + x^2}) \, dx \)

(c) \( \int (x - \sec^2 x + \cos x - \frac{1}{2}) \, dx \)

(d) \( \int_{-5}^{5} \frac{x^{50} \sin x}{x^{50} + 1} \, dx \)

(e) \( \int_{0}^{2} \sin(2\pi x) \, dx \)

(f) \( \int_{0}^{1} 15x^4 \sqrt{3x^5 + 8} \, dx \)

(g) \( \int_{-1}^{1} \ln x \, dx \)

(h) \( \int_{-2}^{2} \frac{1}{x} \, dx \)

(i) \( \int \frac{1}{a^2 + x^2} \, dx \) (\( a \) is a constant)

(j) \( \int \tan x \, dx \)

(12) Differentiate the following functions

(a) \( F(x) = \cos(\ln(6x)) \)

(b) \( h(x) = \frac{e^{2x}}{1 + e^{2x}} \)

(c) \( g(x) = \sqrt{\arcsin 2x} \)

(d) \( G(x) = \int_{0}^{2x} \sqrt{t + 1} \, dt \)

(e) \( f(x) = x^{-5} + 2x^{-4} + x^2 + x + 15 \)

(f) \( p(x) = \log_{5}(2x) \)

(g) \( f(x) = x^{\frac{1}{x}} \)

(h) \( g(x) = x^{3/5}(4 - x) \)

(i) \( h(x) = \ln |x| \)

(j) \( t(x) = 2 \sin x - 3 \cos(x) + 5 \tan x - \cot x + \sec x - 6 \csc x - \arcsin x + 5 \arccos 2x + \arctan(-2x^2) \)