TMM002 – PRE-CALCULUS
5-6 Semester Hours/8-10 Quarter Hours

Recommendation: This course should significantly reflect the Mathematical Association of America's Committee on the Undergraduate Program in Mathematics (CUPM) subcommittee, Curriculum Renewal Across the First Two Years (CRAFTY), College Algebra Guidelines and the CUPM Curriculum Guide.

A Pre-Calculus course is an intensive, more accelerated course for students preparing for a traditional calculus sequence and would generally include essential topics covered in both College Algebra and Trigonometry and as such is only recommended for the more able and prepared student.

Pre-Calculus provides students a college level academic experience that emphasizes the use of algebra and functions in problem solving and modeling, where solutions to problems in real-world situations are formulated, validated, and analyzed using mental, paper-and-pencil, algebraic and technology-based techniques as appropriate using a variety of mathematical notation. Students should develop a framework of problem-solving techniques (e.g., read the problem at least twice; define variables; sketch and label a diagram; list what is given; restate the question asked; identify variables and parameters; use analytical, numerical and graphical solution methods as appropriate; determine the plausibility of and interpret solutions).

– Adapted from the MAA/CUPM CRAFTY 2007 College Algebra Guidelines

Students who are preparing to study calculus need to develop conceptual understanding as well as computational skills. Appropriately designed Pre-Calculus courses can enable students to be successful in calculus. Often, creation of an effective Pre-Calculus course requires learning about different curricular and pedagogical approaches and experimenting with how the most promising ones might be adapted for local implementation. No course should have value *only* as a preparation for a subsequent course; it should have intrinsic value on its own as well as offering preparation for further study.

A Pre-Calculus course should develop mathematical thinking and communications skills by incorporating activities that will help all students progress in developing analytical, critical reasoning, problem-solving, and communication skills and acquiring mathematical habits of mind. More specifically, these activities should be designed to advance and measure students' progress in learning to:

- State problems carefully, modify problems when necessary to make them tractable, articulate assumptions, appreciate the value of precise definition, reason logically to conclusions, and interpret results intelligently;
- Approach problem solving with a willingness to try multiple approaches, persist in the face of difficulties, assess the correctness of solutions, explore examples, pose questions, and devise and test conjectures;
- Read mathematics with understanding and communicate mathematical ideas with clarity and coherence through writing and speaking.

A Pre-Calculus course should communicate the breadth and interconnections of the mathematical sciences by:

- Presenting key ideas and concepts from a variety of perspectives;
- Employing a broad range of examples and applications to illustrate and motivate the material;
- Promoting awareness of connections to other subjects (both in and out of the mathematical sciences), and strengthen each student's ability to apply the course material to these subjects;
- Introduce contemporary topics from the mathematical sciences and their applications, and enhance student perceptions of the vitality and importance of mathematics in the modern world.

– Adapted from the MAA/CUPM 2004 Curriculum Guide

The prerequisite for Pre-Calculus is generally by placement or as a minimum a course in Intermediate Algebra (covering topics such as operations and equations with rational expressions, equations of a line, introduction to functions, introduction to systems of linear equations in two or three variables, absolute-value equations and inequalities, rational exponents, operations and equations with radicals, introduction to complex numbers, quadratic equations and various application problems on these topics).

To qualify for TMM002 (Pre-Calculus), a course must cover as a minimum the essential learning outcomes, noted by an asterisk *, which include all the topics under Functions, Equations/Systems, Sequences/Series, More Trigonometry, and Vectors. A course in Pre-Calculus may also commonly include some of the listed nonessential learning outcomes. These optional topics should be included only if there is adequate course time to do so beyond giving primary course attention to the essential learning outcomes. At least 70% of the classroom instructional time has to be spent on the essential learning outcomes. The optional learning outcomes are learning experiences that enhance, reinforce, enrich or are further applications of the essential learning outcomes. If review of prerequisite course content is necessary, only a minimal amount of time should be devoted to such review.

The successful Pre-Calculus student should be able to:

1. **Functions ***

   1.1 Represent functions verbally, numerically, graphically and algebraically, including linear, quadratic, polynomial, rational, root/radical/power, piecewise-defined, exponential, logarithmic, trigonometric and inverse trigonometric functions.*

   1.2 Determine whether an algebraic relation or given graph represents a function.*

   1.3 Perform transformations of functions – translations, reflections and stretching and shrinking.*

   1.4 Perform operations with functions – addition, subtraction, multiplication, division and composition.*

   1.5 Analyze the algebraic structure and graph of a function, including those listed in (1.1), to determine intercepts, domain, range, intervals on which the function is increasing, decreasing or constant, the vertex of a quadratic function, asymptotes, whether the function is one-to-one, whether the graph has symmetry (even/odd), etc., and given the graph of a function to determine possible algebraic definitions.*

   1.6 Find inverses of functions listed in (1.1) and understand the relationship of the graph of a function to that of its inverse.*

   1.7 Use the Remainder and Factor Theorems for polynomial functions.*

   1.8 Use functions, including those listed in (1.1), to model a variety of real-world problem-solving applications.*

2. **Equations/Systems ***

   2.1 Understand the difference between an algebraic equation of one, two or more variables and a function, and the relationship among the solutions of an equation in one variable, the zeros of the corresponding function, and the coordinates of the *x*-intercepts of the graph of that function.*

2.2 Determine algebraically and graphically whether the graph of an equation exhibits symmetry.*

2.3 Solve a variety of equations, including polynomial, rational, exponential, and logarithmic, trigonometric and inverse trigonometric, including equations arising in application problems.*

2.4 Solve a system of linear equations graphically and algebraically by substitution and elimination, and solve application problems that involve systems of linear equations.*

2.5 Identify and express the conics (quadratic equations in two variables) in standard rectangular form, graph the conics, and solve applied problems involving conics.*

2.6 Solve polynomial and rational inequalities graphically and algebraically.*

3. Sequences/Series *

3.1 Represent sequences verbally, numerically, graphically and algebraically, including both the general term and recursively.*

3.2 Write series in summation notation, and represent sequences of partial sums verbally, numerically and graphically.*

3.3 Identify and express the general term of arithmetic and geometric sequences, and find the sum of arithmetic and geometric series.*

4. More Trigonometry *

4.1 Express angles in both degree and radian measure.*

4.2 Define the six trigonometric functions in terms of right triangles and the unit circle.*

4.3 Solve right and oblique triangles in degrees and radians for both special and non-special angles, and solve application problems that involve right and oblique triangles.*

4.4 Verify trigonometric identities by algebraically manipulating trigonometric expressions using fundamental trigonometric identities, including the Pythagorean, sum and difference of angles, double-angle and half-angle identities.*

4.5 Solve a variety of trigonometric and inverse trigonometric equations, including those requiring the use of the fundamental trigonometric identities listed in (4.4), in degrees and radians for both special and non-special angles. Solve application problems that involve such equations.*

5. Vectors *

5.1 Represent vectors graphically in both rectangular and polar coordinates and understand the conceptual and notational difference between a vector and a point in the plane.*

5.2 Perform basic vector operations both graphically and algebraically – addition, subtraction and scalar multiplication.*

5.3 Solve application problems using vectors.*

6. Perform operations with matrices – addition, subtraction, scalar multiplication and matrix multiplication, including applications with matrices. Use matrices to solve systems of linear equations, including the Gaussian and Gauss-Jordan elimination methods, using a matrix inverse to solve a matrix equation, and Cramer's Rule.

7. Model real-world data with functions for prediction and analysis, including determining the appropriateness of a model and using hand-held calculator or computer regression capability.

8. Use the Rational Zeros Theorem and the Fundamental Theorem of Algebra to find the zeros of and factor a polynomial into linear factors over the complex numbers.

9. Solve a nonlinear system of equations graphically and algebraically, including nonlinear systems of equations arising in application problems.

10. Solve a linear and nonlinear system of inequalities, including linear and nonlinear systems of inequalities arising from application problems.

11. Use the Binomial Theorem.

12. Understand how to eliminate the *xy*-term in the general quadratic equation in two variables by the rotation of axes, and identify and graph conics with rotated axes.

13. Graph complex numbers in the complex plane in both rectangular and polar form, perform operations on such numbers – addition, subtraction, multiplication and division, and use DeMoivre's Theorem to find the n^{th} roots of a complex number.

14. Convert points and equations between rectangular and polar form, graph polar functions, solve polar equations, identify and express the conics in standard polar form for graphing, and solve applied problems involving conics in polar form.

15. Identify and graph a curve defined by parametric equations by making a table of values and, when possible, eliminating the parameter.

16. Understand the average rate of change of the graph of a function or equation on an interval, and informally understand the instantaneous rate of change of the graph of a function or equation at a point and its connection to the slope of the graph at the point, including application problems.