Given $f(x)=\sqrt[3]{x^2}+3$ show that $f'(x)=\frac{2}{3\sqrt[3]{x}}$

i) $f(x+h)=\sqrt[3]{(x+h)^2}+3$

ii) $f(x+h)-f(x)=\sqrt[3]{(x+h)^2}-\sqrt[3]{x^2}$

iii) $$\frac{f(x+h)-f(x)}{h}=\frac{\sqrt[3]{(x+h)^2}-\sqrt[3]{x^2}}{h}=\frac{(x+h)^{2/3}-x^{2/3}}{h}$$

$$=\frac{(x+h)^{2/3}-x^{2/3}}{h}\cdot\frac{\left((x+h)^{4/3}+(x+h)^{2/3}(x)^{2/3}+x^{4/3}\right)}{\left((x+h)^{4/3}+(x+h)^{2/3}(x)^{2/3}+x^{4/3}\right)}$$

$$=\frac{(x+h)^{6/3}+(x+h)^{4/3}x^{2/3}+x^{4/3}(x+h)^{2/3}-x^{2/3}(x+h)^{4/3}-(x+h)^{2/3}x^{4/3}-x^{6/3}}{h\left((x+h)^{4/3}+(x+h)^{2/3}(x)^{2/3}+x^{4/3}\right)}$$

$$=\frac{(x+h)^2-x^2}{h\left((x+h)^{4/3}+(x+h)^{2/3}(x)^{2/3}+x^{4/3}\right)}=\frac{\cancel{x^2}+2xh+h^2-\cancel{x^2}}{h\left((x+h)^{4/3}+(x+h)^{2/3}x^{2/3}+x^{4/3}\right)}$$

$$=\frac{\cancel{h}(2x+h)}{\cancel{h}\left((x+h)^{4/3}+(x+h)^{2/3}x^{2/3}+x^{4/3}\right)}=\frac{2x+h}{(x+h)^{4/3}+(x+h)^{2/3}x^{2/3}+x^{4/3}}$$

iv) $$\lim_{h\to 0}\frac{2x+h}{(x+h)^{4/3}+(x+h)^{2/3}x^{2/3}+x^{4/3}}=\frac{2x}{(x+0)^{4/3}+(x+0)^{2/3}x^{2/3}+x^{4/3}}=\frac{2x}{3x^{4/3}}$$

$$=\frac{2}{3x^{1/3}}$$

$$=\frac{2}{3\sqrt[3]{x}}$$