Math 11012 | Intuitive Calculus | Fall 2014

Quiz 9

Name: Key   Sect: ______

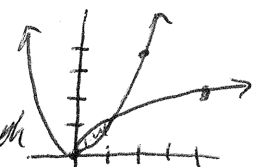
Instructor: Matt Alexander

Quiz Score: ________ /20

Instructions: Please show all reasoning and work using the standard notation correctly.

1. Find the area bounded by the curves $y = x^2$ and $y = \sqrt{x}$

To find our upper:lower bounds we graph



or

$x^2 = \sqrt{x}$

$x^4 = (\sqrt{x})^2$

$x^4 = x$

$x^4 - x = 0$

$x(x^3 - 1) = 0$

$x = 0$ or $x^3 - 1 = 0$

$x^3 = 1$

$x = \sqrt[3]{1} = 1$

$$\int_0^1 (\sqrt{x} - x^2)\,dx = \int_0^1 x^{1/2} - x^2\,dx = \frac{x^{3/2}}{3/2} - \frac{x^3}{3}\Big|_0^1$$

$$= \frac{2}{3}(1)^{3/2} - \frac{1}{3}(1)^3 = \frac{2}{3} - \frac{1}{3} = \boxed{\frac{1}{3}}$$

2. Calculate the Gini Index of income concentration for the Lorenz curve given by $f(x) = \frac{2}{3}x^3 + \frac{x}{3}$ and explain if income is distributed more or less equally for this curve.

$$G.I. = 2\int_0^1 [x - (\tfrac{2}{3}x^3 + \tfrac{x}{3})]\,dx = 2\int_0^1 \left(\frac{2}{3}x - \frac{2}{3}x^3\right)dx = \frac{4}{3}\int_0^1 x - x^3\,dx$$

$$= \frac{4}{3}\left[\frac{x^2}{2} - \frac{x^4}{4}\right]_0^1 = \frac{4}{3}\left(\frac{1}{2} - \frac{1}{4}\right) = \frac{4}{3}\left(\frac{1}{4}\right) = \boxed{\frac{1}{3}}$$

$\frac{1}{3}$ is closer to 0 than 1 and so income is more equally distributed

3. Find the equilibrium point if the demand curve is given by $p = D(x) = 900 - x^2$ and the supply curve is given by $p = S(x) = x^2 + 100$.

$D(x) = S(x)$

$900 - x^2 = x^2 + 100$

$800 = 2x^2$

$400 = x^2$

$x = \pm 20$

We don't want negative demand so...

$x = 20$

$p = S(20) = 20^2 + 100$

$= 500$

So the equilibrium point is (20, 500) or the demand at equilibrium is 20 units at a price of 500 units of money