Math 11022 Summary for Graphs of Trigonometric Functions

In the table, *n* is an integer

function	domain	range	period	"typical" interval for graphing one period, key points & asymptotes	x-intercepts	vertical asymptotes
$f_1(x)=\cos x$	$(-\infty,\infty)$	$[-1,1]$	2π	$[0,2\pi]$ $(0,1),\left(\frac{\pi}{2},0\right),(\pi,-1),\left(\frac{3\pi}{2},0\right),(2\pi,1)$	$x=\frac{\pi}{2}+n\pi$	none
$f_2(x)=\sin x$	$(-\infty,\infty)$	$[-1,1]$	2π	$[0,2\pi]$ $(0,0),\left(\frac{\pi}{2},1\right),(\pi,0),\left(\frac{3\pi}{2},-1\right),(2\pi,0)$	$x=n\pi$	none
$f_3(x)=\sec x$	$x\neq\frac{\pi}{2}+n\pi$	$(-\infty,-1]\cup[1,\infty)$	2π	$(0,2\pi)$ $(0,1),\ x=\frac{\pi}{2},(\pi,-1),\ x=\frac{3\pi}{2},(2\pi,1)$	none	$x=\frac{\pi}{2}+n\pi$
$f_4(x)=\csc x$	$x\neq n\pi$	$(-\infty,-1]\cup[1,\infty)$	2π	$(0,2\pi)$ $x=0,\left(\frac{\pi}{2},1\right),\ x=\pi,\left(\frac{3\pi}{2},-1\right),\ x=2\pi$	none	$x=n\pi$
$f_5(x)=\tan x$	$x\neq\frac{\pi}{2}+n\pi$	$(-\infty,\infty)$	π	$\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ $x=-\frac{\pi}{2},\left(-\frac{\pi}{4},-1\right),(0,0),\left(\frac{\pi}{4},1\right),\ x=\frac{\pi}{2}$	$x=n\pi$	$x=\frac{\pi}{2}+n\pi$
$f_6(x)=\cot x$	$x\neq n\pi$	$(-\infty,\infty)$	π	$(0,\pi)$ $x=0,\left(\frac{\pi}{4},1\right),\left(\frac{\pi}{2},0\right),\left(\frac{3\pi}{4},-1\right),\ x=\pi$	$x=\frac{\pi}{2}+n\pi$	$x=n\pi$

Transformations

$g_1(x)=a\cos(bx-c)+d$

$g_2(x)=a\sin(bx-c)+d$

$g_3(x)=a\sec(bx-c)+d=a\frac{1}{g_1(x)}+d$

$g_4(x)=a\csc(bx-c)+d=a\frac{1}{g_2(x)}+d$

amplitude (vertical stretch/shrink): $|a|$

i) If $a<0$, reflection in the x-axis

ii) Amplitude *only* applies to $g_1(x), g_2(x)$

vertical shift: d up/down

period (horizontal stretch/shrink): $\frac{2\pi}{b}$ **

i) ** For $g_5(x)=a\tan(bx-c)+d$, $g_6(x)=a\cot(bx-c)+d$,

period is $\frac{\pi}{b}$

ii) If $b<0$, use even/odd properties to evaluate the negative

phase (horizontal) shift: $\frac{c}{b}$ right/left