Math 11022
Worksheet 2.2

Key

Trigonometry

Spring 2015
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Instructions: Verify the following identities.

1. $\dfrac{\sin^2\theta+\cos^2\theta}{\cos^2\theta\sec^2\theta}=1$

$$\frac{\sin^2\theta+\cos^2\theta}{\cos^2\theta\sec^2\theta}=\frac{1}{\cos^2\theta\sec^2\theta}=\frac{1}{\cos^2\theta\frac{1}{\cos^2\theta}}=\frac{1}{1}=1$$

2. $2\csc^2\beta=\dfrac{1}{1-\cos\beta}+\dfrac{1}{1+\cos\beta}$

$$\frac{1}{1-\cos\beta}+\frac{1}{1+\cos\beta}=\frac{1+\cos\beta}{(1-\cos\beta)(1+\cos\beta)}+\frac{1-\cos\beta}{(1-\cos\beta)(1+\cos\beta)}=\frac{1+\cos\beta+1-\cos\beta}{1-\cos\beta+\cos\beta-\cos^2\beta}=$$

$$=\frac{2}{1-\cos^2\beta}=\frac{2}{\sin^2\beta}=2\csc^2\beta$$

3. $\dfrac{\sec^2\theta-1}{\sec^2\theta}=\sin^2\theta$

$$\frac{\sec^2\theta-1}{\sec^2\theta}=\frac{\sec^2\theta}{\sec^2\theta}-\frac{1}{\sec^2\theta}=1-\cos^2\theta=\sin^2\theta$$

4. $(\sec^2 x-1)(\sin^2 x-1)=-\sin^2 x$

$$(\sec^2x-1)(\sin^2x-1)=(\tan^2x)(-\cos^2x)=\frac{\sin^2x}{\cos^2x}(-\cos^2x)=-\sin^2x$$

5. $\csc x-\sin x=\cos x\cot x$

$$\csc x-\sin x=\frac{1}{\sin x}-\frac{\sin x}{1}=\frac{1}{\sin x}-\frac{\sin^2x}{\sin x}=\frac{1-\sin^2x}{\sin x}=\frac{\cos^2x}{\sin x}$$

$$=\cos x\cdot\frac{\cos x}{\sin x}=\cos x\cot x$$

6. $\csc(x) + \cot(x) = \dfrac{\sin(x)}{1 - \cos(x)}$

$$\frac{\sin(x)}{1-\cos(x)} = \frac{\sin(x)}{1-\cos(x)} \cdot \frac{1+\cos(x)}{1+\cos(x)} = \frac{\sin(x)(1+\cos(x))}{1-\cos^2(x)} = \frac{\sin(x)(1+\cos(x))}{\sin^2 x}$$

$$= \frac{1+\cos(x)}{\sin(x)} = \frac{1}{\sin(x)} + \frac{\cos(x)}{\sin(x)} = \csc(x) + \cot(x)$$

7. $\dfrac{\tan^2\theta}{1+\sec\theta} = \dfrac{1-\cos\theta}{\cos\theta}$

LHS: $\dfrac{\tan^2\theta}{1+\sec\theta} = \dfrac{\sec^2\theta - 1}{1+\sec\theta} = \dfrac{(\sec\theta+1)(\sec\theta-1)}{1+\sec\theta} = \sec\theta - 1$

RHS: $\dfrac{1-\cos\theta}{\cos\theta} = \dfrac{1}{\cos\theta} - \dfrac{\cos\theta}{\cos\theta} = \sec\theta - 1$

8. $\tan^3 x = \tan x \sec^2 x - \tan x$

$$\tan x \sec^2 x - \tan x = \tan(x)\left[\sec^2 x - 1\right] = \tan(x)\left[\tan^2(x)\right]$$
$$= \tan^3(x)$$

9. $\sin^3 x \cos^4 x = \left(\cos^4 x - \cos^6 x\right)\sin x$

$$\left(\cos^4(x) - \cos^6(x)\right)\sin(x) = \cos^4(x)\left[1-\cos^2(x)\right]\sin(x) = \cos^4(x)\left[\sin^2(x)\right]\sin(x)$$
$$= \sin^3(x)\cos^4(x)$$

10. $\sqrt{\dfrac{1+\sin\theta}{1-\sin\theta}} = \dfrac{1+\sin\theta}{|\cos\theta|}$

$$\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} = \sqrt{\frac{(1+\sin\theta)(1+\sin\theta)}{(1-\sin\theta)(1+\sin\theta)}} = \sqrt{\frac{(1+\sin\theta)^2}{1-\sin^2\theta}} = \sqrt{\frac{(1+\sin\theta)^2}{\cos^2\theta}} = \frac{1+\sin\theta}{|\cos\theta|}$$