| Math 12002 | Analytic Geometry and Calc I | Fall 2016 |
|---|---|---|
| October 4, 2016 | Exam 2 | Matt Alexander |

Name: _____ Score: _____/100

Please show **all** your work! Answers without supporting work will not be given credit. Write answers in spaces provided. You have 50 minutes to complete this exam.

1. (a) (2 points) State the limit definition of the derivative of a function at a point. (i.e. $f'(a)$)

   (b) (8 points) If $f(x) = x^2 + 3x - 5$, find $f'(-6)$ using only the limit definition of the derivative.

   (c) (5 points) Find the equation of the tangent line to the graph of $f(x) = x^2 + 3x - 5$ at the point $(-6, 13)$ in slope intercept form.

   Answer: _____

2. (5 points) Let $y = f(x)$ be a function of $x$. Explain why the instantaneous rate of change of $y$ with respect to $x$ at the point $x = a$ is given by the derivative of $f$ at $a$. (In particular, discuss average rate of change and its relationship to instantaneous rate of change).

3. (24 points) Find the derivatives of the following functions. *Do not simplify your answers.* Clearly mark your answer.

   (a) $f(x) = 8x^4 + \sqrt[4]{x^3} + \dfrac{2}{x^3} + 2^2$

   (b) $g(x) = (3x^2 + x)\tan(x)$

   (c) $h(x) = \dfrac{5 + \cos x}{x^4 + \csc x}$

   (d) $F(x) = (8x^3 + \cos x)^{10}$

4. (7 points) Find the second derivative, $f''$, of $f(x) = (5x + 4)^4$.

Answer:_____

5. (5 points) Let $f$ and $g$ be two differentiable functions such that $f(2) = 3$, $f'(2) = 5$, $g(2) = 6$, and $g'(2) = -1$. If $h(x) = 2f(x)g(x) - g(x)^2$ find $h'(2)$.

Answer:_____

Cont.

6. (7 points) Find the slope of the line tangent to the ellipse $9x^2 + 16y^2 = 25$ at the point $(1, 1)$.

Answer:_____

7. (7 points) Find $y'$ (the derivative of $y$ with respect to $x$) if $3x + y^2 + \cos y + x^2y^3 = 2$.

Answer:_____

8. (10 points) A particle moves along a straight line and its position (in feet) at time $t$ (in seconds) is given by $s(t) = 5t^2 - 40t + 60$.

   (a) Find the velocity and acceleration of the particle at time $t = 3$

v(t)=_____

a(t)=_____

   (b) Determine the time $t$ at which the particle is **not** moving.

Answer:_____

Cont.

9. (10 points) A large spherical balloon is being inflated with helium at a rate of 200 cubic meters of air per minute. Let $r$ be the radius of the balloon, $V$ be its volume, and $t$ be time in minutes. (**Note:** $V = \frac{4}{3}\pi r^3$)

   (a) What is the rate of change of the radius of the balloon when the radius is 2 meters?

   Answer:_____

   (b) What is the rate of change of the radius of the balloon when the radius is 10 meters?

   Answer:_____

   (c) Using complete sentences, compare these two values give an explanation for why they are different.

Cont.

10. (10 points) A ladder 15 feet long rests against a vertical wall. If the bottom of the ladder is pulled away from the wall at a speed of 3 feet per second, how fast is the top of the ladder sliding down the wall when the base is 9 feet from the wall? Be sure to include a diagram, state your variables, and justify your work.

Answer:_____

The End.