Math 12002	Analytic Geometry and Calc I	Fall 2016
October 20, 2016	Exam 3	Matt Alexander

Name: _____ Score: _____/100

Please show **all** your work! Answers without supporting work will not be given credit. Write answers in spaces provided. You have 60 minutes to complete this exam.

1. (10 points) If $f'(x) \leq 10$ for all x and $f(2) = 7$, what is the largest possible value of $f(5)$?

2. (8 points) Does there exist a function f such that $f'(x) \leq 3$ for all x, $f(-2) = 150$, and $f(3) = 200$? Justify your answer.

3. (8 points) Explain, using either Rolle's theorem or MVT, why a ball launched into the air that falls back to the ground must have a time during its flight when its vertical speed is 0.

4. (10 points) Let $f(x) = x^5 + 5x^4 + 5x^3$. Find all critical points, intervals where f is increasing, intervals where f is decreasing, and the x values of all local maxima and minima.

5. (10 points) Let $g(x) = x^3 - 2x^4$. Find all intervals where g is concave up, intervals where g is concave down, and the x values of all points of inflection.

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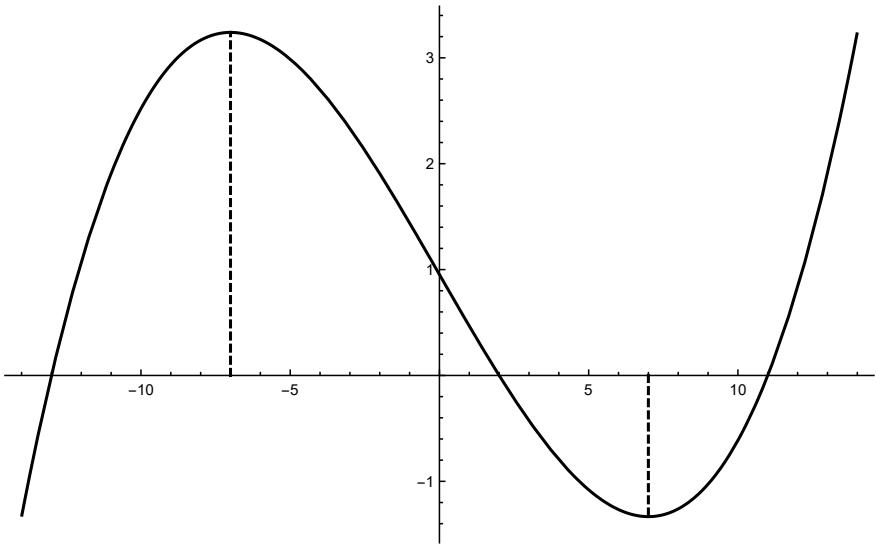
6. (14 points) The graph below is the graph of the DERIVATIVE, $f'(x)$, of a function $y = f(x)$



[NOTE: The graph above is of the derivative f' of f. The questions below refer to f, not to f'.]

(a) Determine the intervals where f is increasing and where f is decreasing.

(b) Determine the intervals where f is concave up and where f is concave down.

(c) Find the x values of all local maxima and minima of f (State whether each is a local maximum or local minimum.)

(d) Find the x values of all inflection points of f.

7. (14 points) Let $f(x) = \frac{4(x^2+3x+1)}{(x+1)^2}$, so that $f'(x) = \frac{4(1-x)}{(x+1)^3}$, and $f''(x) = \frac{8(x-2)}{(x+1)^4}$.
Then $\lim_{x\to+\infty} f(x) = 4$, $\lim_{x\to-\infty} f(x) = 4$, and $\lim_{x\to-1} f(x) = -\infty$, and the following points are on the graph: $\{(1,5), (2,4.8), (0,4), (-2.6,0), (-0.38,0)\}$ (Take the values as I've rounded them)

   (a) Given that f is increasing on $(-1,1)$ and decreasing on $(-\infty,-1)\cup(1,\infty)$, draw the appropriate sign chart for f'.

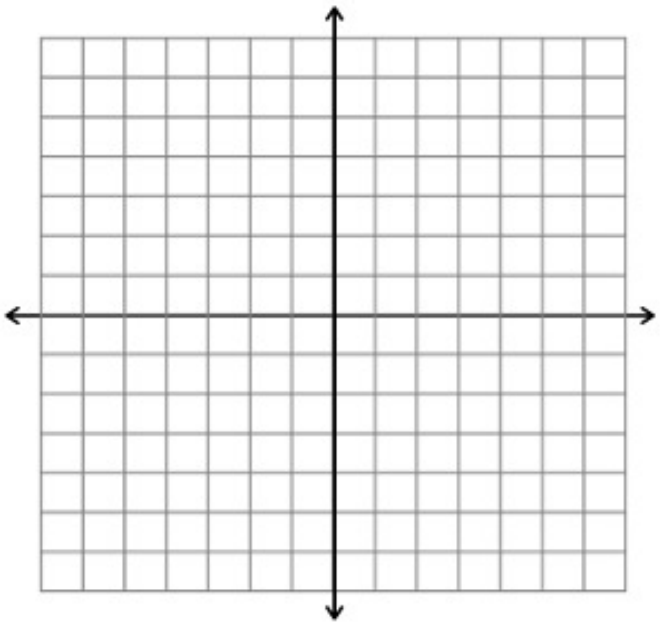
   (b) Find all local maxima and minima of f.

   (c) Given that f is concave-up on $(2,\infty)$ and concave-down on $(-\infty,-1)\cup(-1,2)$, draw the appropriate sign chart for f''.

   (d) Find all inflection points of f.

   (e) Determine the vertical and horizontal asymptotes of f, if any. Justify your answer by the given information.

   (f) Sketch the graph of f on the set of axes below, clearly indicating all of the information given and obtained above.



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8. (10 points) Find the absolute maximum and absolute minimum values of the function $f(x) = x^3 - 6x^2$ on the closed interval $[-1, 7]$

9. (10 points) If the product of two positive numbers is 9, what is smallest possible value of the sum of their squares?

10. (6 points) If a box with a square bottom and open top is to be made from 48 sq. ft. of material, find the function $V(x)$ that gives the volume of the box in terms of one side length. [Note: This is asking you to set up the beginning of an optimization problem. Do not solve]

The End.