Math 12002 | Analytic Geometry and Calc I | Fall 2016

December 8, 2016 | Exam 5 | Matt Alexander

Name: Key Score: /100

Please show **all** your work! Answers without supporting work will not be given credit. Write answers in spaces provided. You have 50 minutes to complete this exam.

1. (6 points) Let $f(x) = e^{4x} + \arccos x$. Find the equation of the tangent line to the graph of f at the point $(0, 1)$.

$f'(x) = 4e^{4x} - \frac{1}{\sqrt{1-x^2}}$

$f'(0) = 4 - 1 = 3 = m$

$y - 1 = 3(x - 0)$

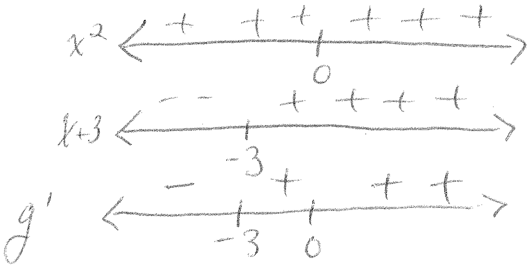
$y = 3x + 1$

Answer: $y = 3x + 1$

2. (6 points) Let $g(x) = x^3 e^x$. Determine the intervals where g is increasing and the intervals where g is decreasing.

$g'(x) = 3x^2e^x + x^3e^x$

$= x^2(x+3)e^x$



Answer: Dec: $(-\infty, -3)$ Inc $(-3, 0) \cup (0, \infty)$

3. (6 points) Let $h(x) = xe^x$. Determine the intervals where h is concave up and the intervals where h is concave down.

$h'(x) = e^x + xe^x$

$h''(x) = e^x + e^x + xe^x$

$= 2e^x + xe^x$

$= (x+2)e^x$

Answer: Conc Up $(-2, \infty)$ Down $(-\infty, -2)$

4. (24 points) Find the derivatives of the following functions. DO NOT SIMPLIFY YOUR ANSWERS.

(a) $f(x) = \sec^{-1} x + \frac{1}{x} + e^{-4x}$

Answer: $\frac{1}{x\sqrt{x^2-1}} + \frac{-1}{x^2} - 4e^{-4x}$

(b) $f(x) = \ln(\cos(5x))$

Answer: $\frac{-5\sin(5x)}{\cos(5x)} = -5\tan(5x)$

(c) $f(x) = e^{x^3-1}(\tan^{-1} x)^3$

Answer: $3x^2e^{x^3-1}(\tan^{-1}(x))^3 + 3e^{x^3-1}(\tan^{-1}(x))^2 \cdot \frac{1}{x^2+1}$

(d) $f(x) = \frac{\arcsin(x^2+2)}{x^5+e^x}$

Answer: $\frac{\frac{x^5+e^x}{\sqrt{1-(x^2+2)^2}} - (5x^4+e^x)\arcsin(x^2+2)}{(x^5+e^x)^2}$

Cont.

5. (30 points) Compute the following indefinite integrals.

(a) $\int (\sin x)e^{2+3\cos x}\,dx$ $= -\frac{1}{3}\int e^u\,du = -\frac{1}{3}e^u + C =$

Let $u = 2+3\cos(x)$

$du = -3\sin(x)\,dx$

Answer: $-\frac{1}{3}e^{2+3\cos(x)} + C$

(b) $\int \frac{x^4+2}{x^5+10x+8}\,dx$ $= \frac{1}{5}\int \frac{1}{u}\,du = \frac{1}{5}\ln|u| + C =$

Let $u = x^5+10x+8$

$du = (5x^4+10)\,dx$

Answer: $\frac{1}{5}\ln|x^5+10x+8| + C$

(c) $\int \frac{\ln x}{x}\,dx$ $= \int u\,du = \frac{1}{2}u^2 + C =$

Let $u = \ln(x)$

$du = \frac{1}{x}\,dx$

Answer: $\frac{1}{2}(\ln(x))^2 + C$

(d) $\int \frac{1}{\sqrt{5-125x^2}}\,dx$ $= \frac{1}{\sqrt{5}}\int \frac{1}{\sqrt{1-25x^2}}\,dx = \frac{1}{5\sqrt{5}}\int \frac{1}{\sqrt{1-u^2}}\,du = \frac{1}{5\sqrt{5}}\arcsin(u) + C$

Let $u^2 = 25x^2$

$u = 5x$

$du = 5\,dx$

Answer: $= \frac{1}{5\sqrt{5}}\arcsin(5x) + C$

Cont.

6. (6 points) Compute the definite integral $\int_0^{\ln 2} \frac{e^x}{e^x+3}\,dx$. $= \int_{x=0}^{x=\ln(2)} \frac{1}{u}\,du = \ln|u| \Big|_{u=4}^{u=5}$

Let $u = e^x + 3$

$du = e^x\,dx$

Answer: $\ln(4) - \ln(5) = \ln\left(\frac{4}{5}\right)$

7. (6 points) Use logarithmic differentiation to find the derivative of $f(x) = x^{x^2+5x}$

$y = x^{x^2+5x}$

$\ln(y) = (x^2+5x)\ln(x)$

$\frac{y'}{y} = (2x+5)\ln(x) + \frac{x^2+5x}{x}$

$y' = y\left[(2x+5)\ln(x) + (x+5)\right]$

Answer: $f'(x) = \left[(2x+5)\ln(x) + (x+5)\right] x^{x^2+5x}$

8. (8 points) Evaluate the following limits. JUSTIFY YOUR ANSWERS.

(a) $\lim_{x\to 0} \frac{1-e^x}{5x+8\sin(x)}$

$\lim_{x\to 0} 1 - e^x = 0$ So by L'Hôpital

$\lim_{x\to 0} 5x + 8\sin(x) = 0$

$\overset{LH}{=} \lim_{x\to 0} \frac{-e^x}{5+8\cos(x)}$

$= \frac{\lim_{x\to 0} -e^x}{\lim_{x\to 0} 5+8\cos(x)}$

$= \frac{-1}{13}$

Answer: $-\frac{1}{13}$

(b) $\lim_{x\to 0} \frac{\cos(x) - 1}{\sin(x) + 1}$ $= \frac{\lim_{x\to 0} \cos(x) - 1}{\lim_{x\to 0} \sin(x) + 1} = \frac{0}{1} = 0$

Answer: 0

(c) $\lim_{x\to\infty} \frac{5e^{2x}}{x + e^{2x}}$ $\overset{LH}{=} \lim_{x\to\infty} \frac{5\cdot 2e^{2x}}{1+2e^{2x}} = \frac{\infty}{\infty}$ X

$\overset{LH}{=} \lim_{x\to\infty} \frac{5\cdot 2^2 e^{2x}}{2^2 e^{2x}}$

$= \lim_{x\to\infty} 5 = 5$

Answer: 5

9. (8 points) Use L'Hôpital's rule to calculate the following limits.

(a) $\lim_{x\to 0^+} x\ln(x)$ $= 0 \cdot -\infty$

$= \lim_{x\to 0^+} \frac{\ln(x)}{\frac{1}{x}} \overset{LH}{=} \lim_{x\to 0^+} \frac{\frac{1}{x}}{-\frac{1}{x^0}} = \lim_{x\to 0^+} -x = 0$

Answer: 0

(b) Use part (a) to compute $\lim_{x\to 0^+} x^x$

$\lim_{x\to 0^+} x^x = \lim_{x\to 0^+} e^{x\ln(x)} = e^{\lim_{x\to 0^+} x\ln(x)} = e^0 = 1$

Answer: 1

The End.