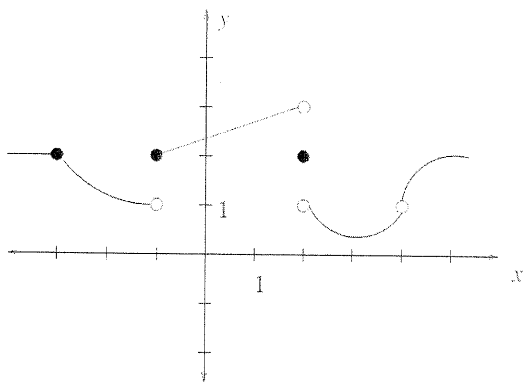
Math 12002 | Analytic Geometry & Calculus I | Fall 2016

Quiz 2 | Matt Alexander

Name: Key | Quiz Score: ______ /10

1. The function f is depicted below. (4 pts)



a) Is f continuous at $x = -3$? Explain.

Yes, $\lim_{x\to -3} f(x) = f(-3) = 2$.

b) Is f continuous at $x = 2$? Explain.

No, $\lim_{x\to 2^+} f(x) = 1$ but $\lim_{x\to 2^-} f(x) = 3$ and so $\lim_{x\to 2} f(x)$ does not exist.

2. Determine whether the following function is continuous at the given value (3 pts).

$$f(x) = \begin{cases} \frac{x^2+x}{x^2+7x+6} & \text{if } x \neq -1 \\ 5 & \text{if } x = -1 \end{cases} \quad \text{at } a = -1$$

- $\lim_{x\to -1} f(x) = \lim_{x\to -1} \frac{x^2+x}{x^2+7x+6} = \lim_{x\to -1} \frac{x(x+1)}{(x+1)(x+6)} = \lim_{x\to -1} \frac{x}{x+6}$, since $x \neq -1$

$$= \frac{\lim_{x\to -1} x}{\lim_{x\to -1}(x+6)} = \frac{-1}{-1+6} = -\frac{1}{5}$$

- $f(-1) = 5$

Hence, since $\lim_{x\to -1} f(x) \neq f(-1)$, f is not continuous at $x = -1$.

3. Use the Intermediate Value Theorem to explain why $f(x) = x^2 - 5$ has a root between 2 and 3.(3 pts).

Since f is a continuous function on the interval $[2, 3]$ (in fact, it is continuous for all real numbers), ~~and~~ $f(2) = -1$, and $f(3) = 4$ we know that there exists a number c between 2 and 3 such that $f(c) = 0$ which is between -1 and 4.