Math 12002 | Analytic Geometry & Calculus I | Fall 2016
Quiz 5 | | Matt Alexander
Name: Key | | Quiz Score: ______ /10

1. Find all of the critical numbers of the function $f(t) = \cancel{t\sqrt{t^2-1}}$ $t\sqrt{1-t^2}$

$$f'(t) = \sqrt{1-t^2} - t^2(1-t^2)^{-\frac{1}{2}} = \frac{1-t^2-t^2}{\sqrt{1-t^2}} = \frac{1-2t^2}{\sqrt{1-t^2}}$$

$f'(t)=0$: $1-2t^2=0$
$t^2 = \frac{1}{2}$
$t = \pm\sqrt{\frac{1}{2}}$

$f'(t)$ DNE: $\sqrt{1-t^2}=0$
$t^2=1$
$t=\pm\sqrt{1}=\pm 1$

Since the domain of $f(t)$ is $[-1,1]$ so the ~~only~~ critical numbers are $\pm\sqrt{\frac{1}{2}}, \pm 1$

2. Give the x values, and the corresponding outputs, of the absolute maximum and minimum values of the function $f(x) = x^3 - 3x + 5$ on the interval $[-1, 3]$.

$f'(x) = 3x^2 - 3 = 3(x^2-1)$

$f'(x)=0$ when $3(x^2-1)=0$
$(x+1)(x-1)=0$
$x=-1$ or $x=1$

$f(-1) = -1+3+5 = 7$
$f(1) = 1-3+5 = 3$
$f(3) = 27-9+5 = 23$

f has ~~a~~ an absolute max of 23 at $x=3$

f has an absolute min of 3 at $x=1$

3. A factory produces widgets. It finds that its profit depends on the number of widgets it produces and can be estimated by $P(x) = \frac{-1}{10}(x^2 - 60x + 100)$ where x is the number of widgets produced per day, and $P(x)$ is the daily profit in hundreds of dollars given x widgets were produced. Their factory can produce between 0 and 100 widgets per day. How many widgets should the factory produce per day? What is their maximum daily profit?

$P'(x) = -\frac{1}{10}(2x-60) = 0$
$x = 30$

$P(30) = -\frac{1}{10}(900-1800+100) = -\frac{1}{10}(-800) = 80$

The widget factory should produce 30 widgets per day for a profit of \$8000.