Math 12002 Analytic Geometry & Calculus I Fall 2016
Quiz 6 Matt Alexander
Name: Key Quiz Score: /10

1. (4pts) If $f(2) = -5$ and $2 \leq f'(x) \leq 5$ for all x.

a) What is the largest possible value of $f(4)$?

By MVT $\exists c$ $f(4)-f(2)=f'(c)(4-2)$

So $f(4)=f(2)+f'(c)(2)=-5+2\cdot f'(c)\leq -5+2\cdot 5=5$ since $f'(c)$

Then $f(4)$ is at most 5.

b) What is the smallest possible value of $f(4)$?

By MVT $\exists c$ $f(4)-f(2)=f'(c)(4-2)$

So $f(4)=f(2)+f'(c)(2)=-5+2\cdot f'(c)\geq -5+2(2)=-1$ since $f'(c)\geq 2$

Then $f(4)$ is at least -1.

2. (3pts) Mr. Colburn is driving along the highway. He gets on the highway at mile marker 0. After 2 hours he is 40 miles down the highway. If his speed never exceeds 45 miles per hour, how far along the highway can Mr. Colburn be after 5 hours? Justify your answer using a theorem.

Let $s(t)$ be Mr. Colburn's distance down the highway at time t (in hours). Then $s'(t)$ is his speed and so by MVT $\exists c$ $s(5)-s(2)=s'(c)(5-2)$ or $s(5)=40+3s'(c)$ since $s'(c)\leq 45$

$s(5)\leq 40+3\cdot 45 = 40+135 = 175$ miles

Mr. Colburn is no more than 175 miles down the highway.

3. (3pts) Does there exist a function f such that $f'(x) > 2$ for all x, $f(2) = 10$, and $f(4) = 11$? Justify your answer.

By MVT $\exists c$ $f(4)-f(2)=f'(c)(4-2)$

or $11-10=f'(c)$

$f'(c)=\frac{f(4)-f(2)}{4-2}=\frac{11-10}{2}=\frac{1}{2}$

but $f'(x)>2$ and so no such c can exist.

Hence the function can't exist