

1. (3pts) Find the critical numbers of $f(x) = x^{\frac{1}{3}}(x+4)$.

$$f'(x) = \frac{1}{3}x^{-\frac{2}{3}}(x+4) + x^{\frac{1}{3}}$$

$$= \frac{\frac{1}{3}(x+4)+x}{x^{\frac{2}{3}}} = \frac{\frac{4}{3}x+\frac{4}{3}}{x^{\frac{2}{3}}} = \left(\frac{4}{3}\right)\left(\frac{x+1}{x^{\frac{2}{3}}}\right)$$

Then $f'(x)=0$ when $x+1=0$
 $x=-1$

and $f'(x)$ DNE when $x^{\frac{2}{3}}=0$

$x=0$

So the critical numbers are $x=-1$ and $x=0$

2. (7pts) Given $f(x) = 2 + 3x - x^3$.

- a) Show that $f'(x) = 3(1+x)(1-x)$.

$$f'(x) = 3 - 3x^2 = 3(1-x^2) = 3(1-x)(1+x)$$

- b) On what intervals is the graph of f increasing and decreasing?

$$\begin{array}{c} f' \\ \leftarrow - + + - \end{array}$$

$-1 \quad 1$

f is increasing on $(-1, 1)$

f is decreasing on $(-\infty, -1) \cup (1, \infty)$

- c) List the local extrema of f .

f has a local min of 0 at $x=-1$

f has a local max of 4 at $x=1$

- d) Show that $f''(x) = -6x$.

$$f''(x) = -6x$$

- e) On what intervals is the graph of f concave up and down?

$$\begin{array}{c} f'' \\ + + - - \\ 0 \end{array}$$

f is concave up on $(-\infty, 0)$

f is concave down on $(0, \infty)$

- f) List the inflection points of f .

f has an inflection point at $(0, 2)$