DERIVATIVE PRACTICE II: SOLUTIONS
I. Exponentials and Logarithms

A. Natural Logarithms

1. $f'(x) = \dfrac{1}{\sin x} \cdot \cos x$

2. $f'(x) = -(\ln x)^{-2} \cdot \dfrac{1}{x}$

3. $f'(x) = \dfrac{1}{x^2} \cdot 2x = \dfrac{2}{x}$

4. $f'(x) = \dfrac{1}{10/x} \cdot (-10x^{-2}) = -\dfrac{1}{x}$

5. $f'(x) = \dfrac{1}{x^{10}} \cdot (10x^9) = 10 \cdot \dfrac{1}{x}$

6. $f'(x) = \dfrac{1}{3x} \cdot 3 + 4 \cdot \dfrac{1}{x}$

7. $f'(x) = 2x \cdot \ln(2x) + x^2 \cdot \dfrac{1}{2x} \cdot 2$

8. $f'(x) = \ln x + x \cdot \dfrac{1}{x}$

9. $f'(x) = \dfrac{1}{1/x} \cdot (-x^{-2}) = -\dfrac{1}{x}$

10. $f'(x) = 3(\ln x)^2 \cdot \dfrac{1}{x}$

11. $f'(x) = \ln(\sqrt{x}) + x \cdot \dfrac{1}{\sqrt{x}} \cdot \dfrac{1}{2}x^{-1/2}$

12. $f'(x) = \dfrac{1}{2}(\ln x)^{-1/2} \cdot \dfrac{1}{x}$

13. $f'(x) = \dfrac{x^2+1}{x-1} \cdot \dfrac{(x^2+1)-(x-1)\cdot 2x}{(x^2+1)^2} = \dfrac{1}{x-1} - \dfrac{1}{x^2+1} \cdot 2x$

14. $f'(x) = \sqrt{\dfrac{x^2+1}{x}} \cdot \dfrac{1}{2}\left(\dfrac{x}{x^2+1}\right)^{-1/2} \cdot \dfrac{(x^2+1)-x(2x)}{(x^2+1)^2} = \dfrac{1}{2}\left[\dfrac{1}{x} - \dfrac{1}{x^2+1} \cdot 2x\right]$

15. $f'(x) = 2x\ln(1-x^2) + x^2 \cdot \dfrac{1}{1-x^2} \cdot (-2x)$

16. $f'(x) = \left(\dfrac{x^2+2}{x^2+3}\right)^{-2/3} \cdot \dfrac{2}{3}\left(\dfrac{x^2+2}{x^2+3}\right)^{-1/3} \cdot \dfrac{2x(x^2+3)-(x^2+2)\cdot 2x}{(x^2+3)^2} = \dfrac{2}{3}\left[\dfrac{1}{x^2+2}\cdot 2x - \dfrac{1}{x^2+3}\cdot 2x\right]$

17. $f'(x) = (\cos(\ln 2x)) \cdot \dfrac{1}{2x} \cdot 2$

18. $f'(x) = \dfrac{\frac{1}{x^2+3} \cdot 2x \cdot \ln(x^3+2) - (\ln(x^2+3)) \cdot \frac{1}{x^3+2} \cdot 3x^2}{(\ln(x^3+2))^2}$

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19. $f'(x) = \dfrac{1}{x^3 - x^2} \cdot (3x^2 - 2x)$

20. $f'(x) = \dfrac{1}{\tan 2x} \cdot (\sec^2 2x) \cdot 2$

B. Natural Exponentials

21. $f'(x) = e^{x^2} \cdot 2x$

22. $f'(x) = (2x + 3)e^x + (x^2 + 3x)e^x$

23. $f'(x) = e^x + xe^x - e^{-x} \cdot (-1)$

24. $f'(x) = \dfrac{e^{x^2} \cdot 2x \cdot e^{x-1} - e^{x^2} \cdot e^{x-1}}{e^{2(x-1)}} = e^{x^2 - x + 1} \cdot (2x - 1)$

25. $f'(x) = e^{\sqrt{x}} \cdot \frac{1}{2}x^{-\frac{1}{2}}$

26. $f'(x) = e^{3x} \cdot 3 + 2e^{2x} \cdot 2 - 3e^x$

27. $f'(x) = e^{x^2 - 2} \cdot 2x$

28. $f'(x) = \dfrac{(e^{2x} \cdot 2)(2 - e^{2x}) - (1 + e^{2x})(-e^{2x} \cdot 2)}{(2 - e^{2x})^2}$

29. $f'(x) = e^{3x-1} \cdot 3 - 4e^{-x} \cdot (-1)$

30. $f'(x) = (-\sin(e^x)) \cdot e^x$

31. $f'(x) = 3e^{2x} \cdot 2 - 4e^x$

32. $f'(x) = e^{3\cos(2x)}(-3\sin(2x)) \cdot 2$

33. $f'(x) = e^{-2x} \cdot (-2) + 4e^{-3x} \cdot (-3)$

34. $f'(x) = e^{2x+1} \cdot 2$

35. $f'(x) = e^{\sin x} \cdot \cos x$

36. $f'(x) = e^{2x} \cdot 2$

37. $f'(x) = \dfrac{-e^{-x}}{(1 - e^{-x})^2}$

38. $f'(x) = \dfrac{(e^{-x} \cdot (-1)) \cdot x - e^{-x}}{x^2}$

39. $f'(x) = 2xe^{-x} + x^2 e^{-x} \cdot (-1)$

40. $f'(x) = e^{-1/x^2} \cdot 2x^{-3}$

41. $f'(x) = e^{\sqrt{x^2+1}} \cdot \frac{1}{2}(x^2 + 1)^{-1/2} \cdot 2x$

42. $f'(x) = \dfrac{e^{2x} \cdot 2 - e^{-x} \cdot (-1)}{2}$

43. $f'(x) = (e^{2x} \cdot 2)\tan(3x) + e^{2x}(\sec^2(3x)) \cdot 3$

44. $f'(x) = e^{\sec(4x)}(\sec(4x)\tan(4x)) \cdot 4$

45. $f'(x) = \dfrac{(e^x + e^{2x} \cdot 2)(e^{3x} + e^{4x}) - (e^x + e^{2x})(e^{3x} \cdot 3 + e^{4x} \cdot 4)}{(e^{3x} + e^{4x})^2}$

C. General Exponentials

46. $f'(x) = 2x \cdot 2^x + x^2 \cdot (\ln 2) \cdot 2^x$

47. $f'(x) = (\ln 3) \cdot 3^{5x} \cdot 5$

48. $f'(x) = 4x^3 + (\ln 4) \cdot 4^x$

49. $f'(x) = (\ln 9) \cdot 9^{-x} \cdot (-1)$

50. $f'(x) = (\sec^2(5^x)) \cdot (\ln 5) \cdot 5^x$

51. $f'(x) = (\ln 3) \cdot 3^{4x+1} \cdot 4 + (\ln 2) \cdot 2^{4x+2} \cdot 4$

52. $f'(x) = (\ln 3) \cdot 3^{x^2+1} \cdot 2x$

53. $f'(x) = (\ln 2) \cdot 2^x$

54. $f'(x) = (\ln 2) \cdot 2^{-x} \cdot (-1)$

55. $f'(x) = \left(\ln \frac{1}{2}\right) \cdot \left(\frac{1}{2}\right)^x$

56. $f'(x) = (\ln 7) \cdot 7^{x^4} \cdot 4x^3$

57. $f'(x) = (\ln 3) \cdot 3^x \tan x + 3^x \sec^2 x$

58. $f'(x) = \dfrac{(2x + (\ln 2)2^x)(x^e + e^x) - (x^2 + 2^x)(ex^{e-1} + e^x)}{(x^e + e^x)^2}$

59. $f'(x) = 35x^6 \cdot 5^x + (5x^7 + 3) \cdot (\ln 5) \cdot 5^x$

60. $f'(x) = x^{3x}(3 \ln x + 3x \cdot \frac{1}{x})$

61. $f'(x) = x^{x^3+5}[3x^2 \ln x + (x^3 + 5) \cdot \frac{1}{x}]$

62. $f'(x) = (\sin x)^x[\ln(\sin x) + x \cdot \frac{1}{\sin x} \cdot \cos x]$

63. $f'(x) = (\tan x)^{\sin x}[\cos x \cdot \ln(\tan x) + \sin x \cdot \dfrac{1}{\tan x} \cdot \sec^2 x]$

64. $f'(x) = (x^e)^x(e \ln x + ex \cdot \frac{1}{x})$

65. $f'(x) = (x^4 + 3)^x[\ln(x^4 + 3) + x \cdot \frac{1}{x^4+3} \cdot 4x^3]$

II. Inverse Trig Functions

66. $f'(x) = \dfrac{1}{\sqrt{1 - (2x)^2}} \cdot 2$

67. $f'(x) = \dfrac{1}{1 + (x^2)^2} \cdot 2x$

68. $f'(x) = \operatorname{arcsec} x + x \cdot \dfrac{1}{x\sqrt{x^2 - 1}}$

69. $f'(x) = \dfrac{1}{\sqrt{1 - \left(\frac{2}{x}\right)^2}} \cdot (-2x^{-2})$

70. $f'(x) = \dfrac{1}{(2x - 3)\sqrt{(2x - 3)^2 - 1}} \cdot 2$

71. $f'(x) = 2\arctan x + 2x \cdot \dfrac{1}{1 + x^2}$

72. $f'(x) = \dfrac{1}{1 + (5x)^2} \cdot 5$

73. $f'(x) = \dfrac{1}{(x^4 + 3)\sqrt{(x^4 + 3)^2 - 1}} \cdot 4x^3$

74. $f'(x) = \dfrac{1}{1 + (3x - 4)^2} \cdot 3$

75. $f'(x) = \dfrac{1}{\sqrt{1 - \left(\frac{x}{4}\right)^2}} \cdot \dfrac{1}{4}$

76. $f'(x) = 2(\sin^{-1} x) \cdot \dfrac{1}{\sqrt{1 - x^2}}$

77. $f'(x) = 2x \tan^{-1} x + (1 + x^2) \cdot \dfrac{1}{1 + x^2}$

78. $f'(x) = \dfrac{1}{1 + \sin^2 x} \cdot \cos x$

79. $f'(x) = -(\tan^{-1} x)^{-2} \cdot \dfrac{1}{1 + x^2}$

80. $f'(x) = \dfrac{\frac{1}{\sqrt{1-x^2}} \cdot \sec^{-1} x - (\sin^{-1} x) \cdot \frac{1}{x\sqrt{x^2-1}}}{(\sec^{-1} x)^2}$

III. General Combinations

81. $f'(x) = e^x \ln x + e^x \cdot \dfrac{1}{x}$

82. $f'(x) = e^{(x^2 + \ln x)} \cdot \left(2x + \dfrac{1}{x}\right)$

83. $f'(x) = \dfrac{e^{(x^2)} \cdot 2x \cdot \ln(x^2 + 5) - e^{(x^2)} \cdot \frac{1}{x^2+5} \cdot 2x}{(\ln(x^2 + 5))^2}$

84. $f'(x) = \dfrac{1}{3xe^{-x}} \cdot [3e^{-x} + 3xe^{-x} \cdot (-1)] = \dfrac{1}{x} - 1$

85. $f'(x) = \dfrac{1 + e^x}{e^x} \cdot \dfrac{e^x(1 + e^x) - e^x(e^x)}{(1 + e^x)^2} = 1 - \dfrac{1}{1 + e^x} \cdot e^x$

86. $f'(x) = \dfrac{1}{e^{\sin 2x}} \cdot e^{\sin 2x} \cdot (\cos 2x) \cdot 2 = (\cos 2x) \cdot 2$

87. $f'(x) = \dfrac{1}{e^x + 2x} \cdot (e^x + 2)$

88. $f'(x) = x^{\ln x} \cdot 2(\ln x) \cdot \dfrac{1}{x}$

89. $f'(x) = x^{(e^x)} \left[e^x \ln x + e^x \cdot \dfrac{1}{x} \right]$

90. $f'(x) = x^{\tan^{-1} x} \left[\dfrac{1}{1 + x^2} \cdot \ln x + (\tan^{-1} x) \cdot \dfrac{1}{x} \right]$

91. $f'(x) = \dfrac{1}{x} \cdot \sin^{-1} x + (\ln x) \cdot \dfrac{1}{\sqrt{1 - x^2}}$

92. $f'(x) = \dfrac{1}{\sin^{-1} x} \cdot \dfrac{1}{\sqrt{1 - x^2}}$

93. $f'(x) = (e^{3x} \cdot 3) \sin^{-1}(5x) + e^{3x} \cdot \dfrac{1}{\sqrt{1 - (5x)^2}} \cdot 5$

94. $f'(x) = e^x(\sin x)(\sin^{-1} x) + e^x(\cos x)(\sin^{-1} x) + e^x(\sin x) \cdot \dfrac{1}{\sqrt{1 - x^2}}$

95. $f'(x) = e^{\tan^{-1} x} \cdot \dfrac{1}{1 + x^2}$

96. $f'(x) = \dfrac{\left(2x + \frac{1}{x\sqrt{x^2-1}}\right)(x^2 + \sec x) - (x^2 + \sec^{-1} x)(2x + \sec x \tan x)}{(x^2 + \sec x)^2}$

97. $f'(x) = \dfrac{1}{1 + (\ln x)^2} \cdot \dfrac{1}{x}$

98. $f'(x) = \dfrac{1}{e^x\sqrt{(e^x)^2 - 1}} \cdot e^x = \dfrac{1}{\sqrt{e^{2x} - 1}}$

99. $f'(x) = \dfrac{\left(\frac{1}{3x} \cdot 3 + 4e^{4x}\right) \tan^{-1}(5x) - (\ln(3x) + e^{4x}) \cdot \frac{1}{1+(5x)^2} \cdot 5}{(\tan^{-1}(5x))^2}$

100. $f'(x) = \sec^{-1}(1 + e^x) + x \cdot \dfrac{1}{(1 + e^x)\sqrt{(1 + e^x)^2 - 1}} \cdot e^x$