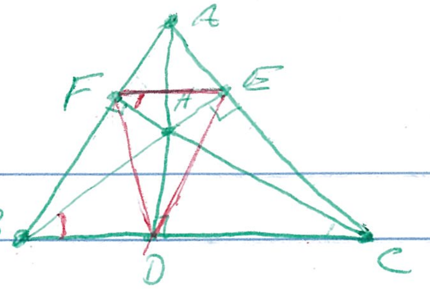

9.3

a) Show BFEC is cyclic

Notice $\angle FHB \cong \angle EHC$ and so $\Delta FHB \sim \Delta EHC$ by AA

Then $\frac{FH}{EH} = \frac{BH}{CH} = k$ or $\frac{CH}{EH} = \frac{BH}{FH} = k$ and again by Vertical angles $\angle FHE \cong \angle BHC$. Then we may use SAS for similar triangles to see that $\Delta HEF \sim \Delta HCB$. Hence $\angle EFH \cong \angle HBD$.

This is enough to show BFEC is cyclic on its own, but to use the equivalent property that we know we must finish by using algebra to show that $\angle EBD + \angle FEC = 180°$.

b) Show $\angle FEC = 180 - \angle ABC$. Done from above

c) Show $\Delta ABC \sim \Delta AEF \sim \Delta DEC \sim \Delta DBF$

We will show $\Delta ABC \sim \Delta AEF$, the rest follow from similar arguments. Clearly $\angle BAC \cong \angle FAE$

Since $\angle FEC + \angle FEA = 180 = \angle FEC + \angle ABC$

we have $\angle ABC \cong \angle FEA$ so by AA $\Delta AEF \sim \Delta ABC$

d) By symmetric arguments to those above with points AFDC, we have AFDC is cyclic

$\angle CAD \cong \angle CFD$ as they subtend the same arc.

Further, $\angle EAH \cong \angle HBD$ since $\Delta AEH \sim \Delta BDH$

So then $\angle EFH \cong \angle EBC \cong \angle HBD \cong \angle EAH \cong \angle CAD \cong \angle CFD$