Math 45021
Proof writing example from informal to formal

Suppose we are given the following theorem and asked to prove it.

Theorem 1. *If $\triangle ABC$ and $\triangle DEF$ are such that $\angle A \cong \angle D, \angle B \cong \angle E$, and $\overline{BC} \cong \overline{EF}$, then $\triangle ABC \cong \triangle DEF$.*

Least formal proof:

Statement	Reasoning
$\angle A \cong \angle D, \angle B \cong \angle E, \& \overline{BC} \cong \overline{EF}$	Given
$AB > DE$	Assumption
$\exists G$ on $\overline{AB}$ with $\overline{BG} \cong \overline{ED}$	$AB > DE$
$\triangle GBC \cong \triangle DEF$	By SAS
$\angle BGC \cong \angle D \cong \angle A$	Since $\triangle GBC \cong \triangle DEF$ and given
$\angle BGC > \angle A$	By exterior angle theorem
Contradiction	$\angle BGC > \angle A$ and $\angle BGC \cong \angle A$
$AB \leq DE$	Contradiction found in assumption
$AB = DE$	Symmetry of argument
$\triangle ABC \cong \triangle DEF$	ASA
We have shown what was desired.	

Construct sentences:

1. It is given that $\angle A \cong \angle D, \angle B \cong \angle E, \& \overline{BC} \cong \overline{EF}$.
2. Let's assume that $AB > DE$.
3. There exists G on $\overline{AB}$ with $\overline{BG} \cong \overline{ED}$.
4. By SAS we have $\triangle GBC \cong \triangle DEF$.
5. This gives $\angle BGC \cong \angle D \cong \angle A$.
6. By Exterior Angle Theorem $\angle BGC > \angle A$.
7. This is a contradiction since $\angle BGC = \angle A$ and $\angle BGC > \angle A$.
8. $AB = DE$ since our argument is symmetric for $AB < DE$.
9. By ASA $\triangle ABC \cong \triangle DEF$ $\square$

Construct paragraphs:

We are given two triangles, $\triangle ABC$ and $\triangle DEF$, with $\angle A \cong \angle D, \angle B \cong \angle E$, and $\overline{BC} \cong \overline{EF}$. From this we want to show that $\triangle ABC \cong \triangle DEF$.

Suppose $AB \ncong DE$. Then without loss of generality we may assume $AB > DE$. We can then say there exists G on BA such that $BG \cong ED$. By SAS $\triangle GBC \cong \triangle DEF$. Thus $\angle BGC \cong \angle D \cong \angle A$, but by the exterior angle theorem $\angle BGC > \angle A$.

Hence we contradict our assumption that $AB \ncong DE$. So $AB = DE$ and by ASA $\triangle ABC \cong \triangle DEF$ as desired. $\square$